ESSENTIAL MECHANICAL PROPERTIES

Elastic and Plastic Deformation

Under enough stress or heat any material, no matter its form, will stretch, squash, twist, or break. Here are some samples that have been tested to failure; from left to right:

- A welded steel bar under 83,400 pounds of tension.
- A copper rod under 119,000 pounds of tension.
- A steel bridge member under 242,000 pounds of compression.
- An aluminum water pipe under 206,500 pounds of compression.
- A laminated yellow pipe timber under 73,500 pounds of compression.

(Photo by the author at the Smithsonian in Washington DC, 1996)

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ESSENTIAL MECHANICAL PROPERTIES

Elastic and Plastic Deformation

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"If all the seals had leaked, it would have been obvious even to NASA that the problem was serious. But only a few of the seals leaked on only some of the flights. So NASA had developed a peculiar kind of attitude: if one of the seals leaks a little and the flight is successful, the problem isn't so serious. Try playing Russian roulette that way: you pull the trigger and the gun doesn't go off, so it must be safe to pull the trigger again...."

Richard Feynman (Nobel Laureate, 1965)
[Richard Feynman talking about the O-ring seal problem that lead to the space shuttle Challenger disaster on January 28, 1986]

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1 Stress, Strain and Elastic Moduli

Consider a solid rod with length $L_o$, diameter $D_o$ and cross sectional area $A_o$, as depicted in Figure 1-1 (a). When tensile forces, those stretching the solid, are applied to this specimen, the rod becomes extended, or stretched as shown in Figure 1-1 (b). At the same time it becomes narrower. Its new length, diameter and cross sectional area are $L$, $D$ and $A$.

\begin{align*}
\text{(a) A body in equilibrium and experiencing no forces. (b) Tensile forces stretch the body to a new length } L, \text{ diameter } D \text{ and cross sectional area } A. \text{ (c) A small increase } \delta F \text{ in the tensile forces gives rise to a small increase } \delta L \text{ in the length, a small change } \delta D \text{ in the diameter and a small change } \delta A \text{ in the cross sectional area.}
\end{align*}

\textbf{Figure 1-1}

The effects of the applied load can only be meaningfully compared between samples if we compare the force experienced per unit cross sectional area of the sample. In other words, we need to compare forces experienced by the same area on different samples. We define \textit{engineering tensile stress} as the applied force per unit original cross sectional area of the sample where the force is perpendicular to the area,

$$\sigma = \frac{F}{A_o} \quad \text{Engineering tensile stress} \quad (1-1)$$

Tensile stresses pull away the faces of application as indicated in Figure 1-1. The forces are at right angles to the faces. If the applied forces are pushing the faces inwards, compressing the sample, then the stresses are said to be \textit{compressive}. The actual stress, or instantaneous stress, is the force per instantaneous unit area. If during the application of the force $F$ the cross sectional area is $A$, as in Figure 1-1 (b), then \textit{true stress} $\sigma_t$ is defined as,

$$\sigma_t = \frac{F}{A} \quad \text{True tensile stress} \quad (1-2)$$

When comparing the amounts of extension different samples exhibit under a given tensile stress, it is useful to compare the extensions per unit original length. If $\Delta L = L - L_o$ is the extension under an applied tensile load then \textit{engineering strain} is defined as,
Engineering strain is based on extensions with respect to the original length \( L_o \). Consider what happens when we increase the tensile load \( F \) by a small amount \( \delta F \) as shown in Figure 1-1 (c). The length \( L \) changes by \( \delta L \). The instantaneous incremental increase in length per unit length \( \delta L / L \) is defined as incremental true strain, \( \delta \varepsilon_t \),

\[
\delta \varepsilon_t = \frac{\delta L}{L} 
\]

Incremental true strain

(1-4)

True strain is then the summation of all incremental true strains from original length, \( L_o \), to final length \( L \),

\[
\varepsilon_t = \int_{L_o}^L \frac{\delta L}{L} = \ln \left( \frac{L}{L_o} \right) 
\]

True strain and engineering strain

(1-5)

or

\[
\varepsilon_t = \ln(1 + \varepsilon) 
\]

From Equation (1-5), it is apparent that for small strains, engineering and true strains are approximately the same (remember that for small \( x \), \( \ln(1+x) \approx x \)). In using engineering stress and strain we typically drop the engineering adjective and simply call these quantities stress and strain whereas true is emphasized for true stress and strain. All experiments show that, provided the strain is small, the strain is proportional to applied stress. This is an elastic behavior in which the strain exhibited by a body increases with the applied stress. If the stress is removed, the strain returns to zero. This type of elastic behavior is called Hooke’s law. The proportionality is usually written as

\[
\sigma = E \varepsilon 
\]

Hooke’s law and Young’s modulus

(1-6)

where \( E \) is an elastic modulus that is called Young’s modulus. Young’s modulus depends on the nature of bonding between the atoms or molecules in the solid inasmuch as the strain is a result of the stretching of bonds between atoms.

A solid that has been strained by an applied stress as in Figure 1-1 (b) is said to be elastically deformed if it returns to its original shape after the removal of the stress. Equation (1-6) is a description of elastic deformation. By convention, a positive stress pulls, or creates tension, and a negative stress pushes, or creates compression. Thus, according to Equation (1-6), compression results in a negative strain or compression as we expect.

When a solid is extended along the direction of an applied tensile stress, along \( z \), as in Figure 1-1 (b) it becomes narrower in the perpendicular directions. The longitudinal stress, \( \sigma_z \), therefore induces not only a longitudinal strain, \( \varepsilon_z \), but also a lateral strain, \( \varepsilon_x \), along \( x \) in a direction perpendicular to \( z \). In Figure 1-1 (b), the lateral strain along \( x \), along any direction perpendicular to \( z \), is defined as \( \varepsilon_x = \Delta D / D_o = (D - D_o) / D_o \) which is a negative quantity for tensile stresses and positive for compressional stresses. The ratio of lateral to longitudinal strain is defined as the Poisson’s ratio, \( \nu \), for that material,

\[
\nu = -\frac{\varepsilon_x}{\varepsilon_z} = -\frac{\text{Lateral strain}}{\text{Longitudinal strength}} 
\]

Poisson’s ratio

(1-7)

The negative sign ensures that \( \nu \) is a positive quantity as it is conventional to list positive numbers as material properties. For many metals, \( \nu \) is typically \( 1/3 \) whereas for polymers and rubbers \( \nu \) is greater than \( 1/3 \).
Shear stresses and strains arise when tangential forces act on surfaces.

**Figure 1-2**

There are other forms of stresses and strains in engineering besides those tensile stresses depicted in Figure 1-1 (b). When opposite forces act tangentially on opposite faces of a brick-like solid as in Figure 1-2, the resulting stresses are called shear stresses as they tend to shear the solid. Shear stress, denoted by $\tau$, is defined as the tangential force per unit surface area on which it acts,

$$\tau = \frac{F_t}{A}$$

*Shear stress* (1-8)

where $F_t$ is the tangential force acting on surface area $A$. Shear stresses cause a solid to become sheared or skewed. In Figure 1-2, shear stresses acting on a brick-type solid induce the opposite surfaces, separated by distance $L$, to be relatively displaced by $\Delta x$ along the forces (along $x$). Greater is $\Delta x$ with respect to $L$, bigger is the extent of skewing. Shear strain, $\gamma$, is defined by

$$\gamma = \frac{\Delta x}{L} = \tan(\theta) = \theta$$

*Shear strain* (1-9)

where $\theta$ is the shearing angle defined in Figure 1-2. As typically $\theta$ is small, $\tan \theta = \theta$ and shear strain is simply this angle of shear. All experiments indicate that for a given material and for small shear strains, the shear deformation is elastic and obeys Hooke's law. $G$ called the shear modulus (also known as modulus of rigidity) relates the shear stress and strain in elastic shear deformation via,

$$\tau = G\gamma$$

*Shear modulus* (1-10)

Another form of stress arises by the uniform application of forces over the whole surface of the body as, for example, when the body is immersed deep into ocean. Force per unit area in this case is called pressure, $P$. 
A body subjected to a hydrostatic pressure is compressed in all directions and exhibits volume strain, $\Delta V/V$.

**Figure 1-3**

### Table 1-1
Types of Elastic Deformation.

<table>
<thead>
<tr>
<th>Stress</th>
<th>Strain</th>
<th>Elastic Modulus</th>
<th>Hooke's Law</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tensile stress, $\sigma$</td>
<td>Tensile strain, $\varepsilon = \Delta L/L_o$</td>
<td>Young's Modulus, $E$</td>
<td>$\sigma = E\varepsilon$</td>
</tr>
<tr>
<td>Shear stress, $\tau$</td>
<td>Shear strain, $\gamma = \theta$</td>
<td>Shear Modulus, $G$</td>
<td>$\tau = G\gamma$</td>
</tr>
<tr>
<td>Hydrostatic Pressure, $P$</td>
<td>Volume strain, $\Delta = \Delta V/V$</td>
<td>Bulk Modulus, $K$</td>
<td>$P = -K\Delta$</td>
</tr>
</tbody>
</table>

Even though pressure is always compressing a body, it is nonetheless, denoted as a positive quantity. Suppose that an applied pressure changes the volume by an amount $\Delta V$ as shown in Figure 1-3. Then volume strain, $\Delta$, is defined by

$$\Delta = \frac{\Delta V}{V}$$

*Volume strain* (1-11)

For small volume strains, the volume deformation is elastic and obeys Hooke’s law with an elastic modulus that is called the *bulk modulus*, $K$:

$$P = -K\Delta$$

*Bulk modulus* (1-12)

The negative sign in Equation (1-12) therefore ensures that the bulk modulus, which is a material property, is tabulated as a positive quantity. Elastic deformation is said to occur *instantaneously*. When a stress is applied to a body, the bonds between atoms become stretched or compressed almost immediately. Furthermore, when the stress is removed, the solid instantaneously\(^1\) and the induced strain occurs returns to its initial state.

---

\(^1\) Elastic deformations travel at the speed of sound so the time it takes to respond to a mechanical stress is the time it takes for the sound to travel down the solid, typically in the range microseconds to milliseconds.
its original shape and size. Elastic deformation is reversible. When we increase the stress from \(\sigma_1\) to \(\sigma_2\), the strain increases from \(\varepsilon_1\) to \(\varepsilon_2\). When the stress is returned back to \(\sigma_1\), the strain also returns back to \(\varepsilon_1\).

Table 1-1 summarizes the three types of elastic moduli encountered above.

### Table 1-2
Typical values of elastic moduli, Poisson’s ratios and melting temperatures for a variety of materials.

<table>
<thead>
<tr>
<th>Material</th>
<th>(E) (GPa)</th>
<th>(\nu)</th>
<th>Melting temperature, (K)</th>
<th>Bonding</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diamond</td>
<td>1,000</td>
<td>0.25</td>
<td>3923</td>
<td>Covalent</td>
</tr>
<tr>
<td>Tungsten</td>
<td>407</td>
<td>0.28</td>
<td>3683</td>
<td>Metallic</td>
</tr>
<tr>
<td>Alumina (Al(_2)O(_3))</td>
<td>390</td>
<td>0.26</td>
<td>1323</td>
<td>Ionic</td>
</tr>
<tr>
<td>Magnesia (MgO)</td>
<td>250</td>
<td>0.163</td>
<td>3098</td>
<td>Ionic</td>
</tr>
<tr>
<td>Nickel</td>
<td>210</td>
<td>0.31</td>
<td>1728</td>
<td>Metallic</td>
</tr>
<tr>
<td>Silicon</td>
<td>190</td>
<td>0.27</td>
<td>1687</td>
<td>Covalent</td>
</tr>
<tr>
<td>Brass</td>
<td>110</td>
<td>0.35</td>
<td>1190</td>
<td>Metallic</td>
</tr>
<tr>
<td>Aluminum</td>
<td>70</td>
<td>0.33</td>
<td>933</td>
<td>Metallic</td>
</tr>
<tr>
<td>Soda glass</td>
<td>69</td>
<td>0.23</td>
<td>900</td>
<td>Ionic</td>
</tr>
<tr>
<td>Nylon</td>
<td>3</td>
<td>0.45</td>
<td>538</td>
<td>Van der Waals</td>
</tr>
<tr>
<td>Polystyrene</td>
<td>3</td>
<td>0.4</td>
<td>513</td>
<td>Van der Waals</td>
</tr>
</tbody>
</table>

Table 1-3 lists some typical values for the elastic moduli of various solids. Solids that have strong interatomic bonding tend to possess large Young's moduli. In polymers, the polymer chains are held together by van der Waals bonds which are weak and therefore easily stretched. Polymers therefore have some of the lowest elastic moduli compared with other solids. We should note that the elastic moduli of homogeneous and isotropic solids are related to each other by relatively simple expressions through Poisson's ratio as indicated in Table 1-3 so that one can be calculated from the other. For example, Young's modulus, \(E\), and shear modulus, \(G\), are related by

\[
E = 2G(1 + \nu)
\]  

Table 1-3
Relationship between Elastic Moduli and Poisson’s ratio for homogeneous and isotropic materials, for example, polycrystalline solids.

<table>
<thead>
<tr>
<th></th>
<th>(E)</th>
<th>(G)</th>
<th>(K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(E)</td>
<td>(E = 2G(1 + \nu))</td>
<td>(E = 3K(1 - 2\nu))</td>
<td></td>
</tr>
<tr>
<td>(G)</td>
<td>(G = \frac{E}{2(1 + \nu)})</td>
<td>(G = \frac{3(1 - 2\nu)}{2(1 + \nu)}K)</td>
<td></td>
</tr>
<tr>
<td>(K)</td>
<td>(K = \frac{E}{3(1 - 2\nu)})</td>
<td>(K = \frac{2(1 + \nu)}{3(1 - 2\nu)}G)</td>
<td></td>
</tr>
</tbody>
</table>

Table 1-2 lists some typical values for the elastic moduli of various solids. Solids that have strong interatomic bonding tend to possess large Young's moduli. In polymers, the polymer chains are held together by van der Waals bonds which are weak and therefore easily stretched. Polymers therefore have some of the lowest elastic moduli compared with other solids. We should note that the elastic moduli of homogeneous and isotropic solids are related to each other by relatively simple expressions through Poisson's ratio as indicated in Table 1-3 so that one can be calculated from the other. For example, Young's modulus, \(E\), and shear modulus, \(G\), are related by

\[
E = 2G(1 + \nu)
\]  

(1-13)
and since \( \nu \approx \frac{1}{3} \) for metals this means, \( E \approx \frac{8}{3} G \). For polycrystalline aluminum, for instance, measurements indicate \( E = 70 \text{ GPa} \) and \( G = 26 \text{ GPa} \) so that \( E \approx \frac{8}{3} G \).

For a homogeneous isotropic solid, the elastic moduli, \( E \), \( G \) and \( K \) are related through

\[
\frac{1}{E} = \frac{1}{3G} + \frac{1}{9K}
\]

The elastic moduli (1-14)

When describing stresses the associated strains and elastic moduli, it is worth remembering that “tension is about pulling, compression is about pushing, and shear is about sliding”.

**Example 1-1: Young's modulus and Poisson's ratio**

Consider a 10 meter steel wire of diameter 3 mm that is carrying a tensile load of 1.2 kN (equivalent to a mass of about 120 kg). If the elastic modulus and Poisson's ratio of steel are 210 GPa and 0.27 what is the new length and diameter of the steel wire assuming that the elongation is elastic? Compare the true and engineering strains and also true and engineering stresses.

**Solution**

The applied tensile stress (engineering stress) along the wire taken as the \( z \)-direction is

\[
\sigma = \frac{F}{A} = \frac{F}{\pi \left(\frac{D}{2}\right)^2} = \frac{1.2 \times 10^3 \text{ N}}{\pi \left(1.5 \times 10^{-3} \text{ m}\right)^2} = 169.77 \text{ MPa}
\]

The resulting engineering strain along \( z \) can be found from \( \sigma = E \varepsilon_z \),

\[
\varepsilon_z = \frac{\sigma}{E} = \frac{169.77 \times 10^6 \text{ Pa}}{210 \times 10^9 \text{ Pa}} = 8.084 \times 10^{-4}
\]

If \( L \) is new length of the wire, then

\[
\varepsilon_z = \frac{L - L_0}{L_0}
\]

which gives \( L = L_0 (1 + \varepsilon_z) = (10 \text{ m})(1 + 8.084 \times 10^{-4}) = 10.0081 \text{ m} \)

The extension \( L - L_0 \) is therefore 8.1 mm nearly 1 cm.

The lateral strain, \( \varepsilon_x \), perpendicular to \( z \), can be found from the Poisson ratio, \( \nu \),

\[
\varepsilon_x = -\nu \varepsilon_z = -(0.27)(8.0841 \times 10^{-4}) = -2.181 \times 10^{-4}
\]

If \( D \) is the new diameter then

\[
\varepsilon_x = \frac{D - D_0}{D_0}
\]

which gives \( D = D_0 (1 + \varepsilon_x) = (3 \text{ mm})(1 - 2.181 \times 10^{-4}) = 2.9993 \text{ mm} \), slightly less than the original diameter as expected.

To find the true stress we need the cross sectional area \( A \) while the force is applied which is \( \pi \left(\frac{1}{2} D\right)^2 \). The true stress is

\[
\sigma_t = \frac{F}{A} = \frac{F}{\pi \left(\frac{D}{2}\right)^2} = \frac{120 \times 10^3 \text{ N}}{\pi \left(\frac{2.993 \times 10^{-3} \text{ m}}{2}\right)^2} = 169.84 \text{ MPa}
\]
Clearly the true stress is only 0.043% different than the engineering stress. The true strain along \( z \) is
\[
\varepsilon_t = \ln(1+\varepsilon) = \ln(1+ 8.084 \times 10^{-4}) = 8.0775 \times 10^{-4}
\]
The true strain is 0.04% smaller than the engineering strain.

**Example 1-2: Compressibility**

Find the pressure that is needed to compress the volume of an aluminum object by 0.1%. At what ocean depth is the pressure equal to this value?

**Solution**

We need the compressibility \( K \) whereas we are given \( E \) and \( \nu \) in Table 1-2. For Al, \( E = 70 \) GPa and \( \nu = 0.33 \) so that from Table 1-3 we have
\[
K = \frac{E}{3(1 - 2\nu)} = \frac{70 \text{ GPa}}{3(1 - 2 \times 0.33)} = 68.63 \text{ GPa}
\]

We should note that since for metals \( \nu \approx 1/3, K \approx E \); and this is apparent in the above calculation.

For a volume strain of \( \Delta = -0.001 \) we have from \( P = -K\Delta \),
\[
P = -(68.63 \times 10^9)(-0.001) = 68.63 \text{ MPa}.
\]

The pressure in the sea at a depth \( h \) is \( \rho gh \) where \( \rho \) is the density of water, 1000 kg m\(^{-3}\), and \( g \approx 10 \text{ m s}^{-2} \) so that \( h \) is about 6.9 km.

2 Stress-Strain Characteristics, Brittle and Ductile Materials

Tensile stress–strain characteristics of a material are typically examined using a tensile test machine. The specimen, in the shape of a rod as depicted in Figure 2-1, is pulled by a progressively increasing tensile load until it is fractured.

The end regions of the rod have a larger diameter and are also threaded for screwing the specimen to the specimen grips in the cross bars of the test apparatus as illustrated in Figure 2-2. This good grip leads to a proper transfer of the tensile load to the specimen. The end regions have a larger diameter to reduce the applied stress in these regions thereby avoiding unwelcomed fracture due to threads or other machining defects in the ends regions.
Tensile stress-strain tests are carried out by applying a tensile load in a tensile test machine.

**Figure 2-2**

![Diagram showing tensile test setup](image)

Typical engineering stress-engineering strain characteristics from a tensile test on a ductile polycrystalline metal (e.g. aluminum alloys, brasses, bronzes, nickel etc.)

**Figure 2-3**

The central region of the specimen has a uniform and narrower cross sectional area. This is the area that becomes deformed during the tensile loading as the stress becomes maximum in this reduced diameter section. Mechanical engineers have established a standard specimen size and geometry for tensile tests which will not be elaborated here. A strain gauge or an extensometer is attached to the central region of the specimen and monitors the extension of the sample under load. The original length of the strain gauge (or the length between the extensometer grips) is $L_o$ and its extension $\Delta L = L - L_o$ is monitored as a function of the applied tensile load $F$ as the specimen is loaded at a constant rate until eventually it fractures. $L_o$ is typically 2 in. (50 mm) and is called the *gauge length*. If $D_o$ is the original cross sectional area of the
As the instantaneous diameter or cross sectional area is conventionally not monitored we cannot normally determine the actual or true stress ($\sigma_f$). Since true and engineering strains are related, $\varepsilon_t = \ln(1+\varepsilon)$, it seems that the true strain can be determined. This is not, however, entirely true as discussed below.

A typical stress vs. strain result for a ductile polycrystalline metal (such as an aluminum alloy) from a tensile test is shown in Figure 2-3. Initially the stress–strain characteristic is linear and represents the elastic deformation of the specimen. The extension of the sample is due to the stretching of the interatomic bonds. As the load is increased, the bonds are stretched even more to balance the applied tensile forces. The slope of this linear region from the origin represents Young’s modulus or elastic modulus, $E$. If we unload the sample anywhere in the linear region, say at point $A$, the sample size and shape return to their original values along $OA$. Thus, after unloading, strain always returns to $O$ in this linear or elastic regime. As the load is increased further eventually a point $Y$ is reached where the deformation is no longer elastic. Beyond point $Y$ the sample yields permanently by deforming plastically under load. Point $Y$ marks the end of the elastic regime and the onset of plastic deformation in which deformation involves material flow and is therefore permanent.

If we were to unload the specimen in this plastic region, say at point $P$, then the sample would recover along a line parallel to the elastic region. This elastic recovery, along $PO'$, represents the stretched bonds recoiling back (springing back) to their equilibrium lengths. But the specimen is left with a permanent plastic deformation $\varepsilon_{pl}$ corresponding to $OO'$. Reapplying the load would elongate the specimen along the line $O'P$ just as if it had started at $O'$. In the plastic deformation region, from point $Y$ onwards, the deformation of the specimen is permanent and is due to material flow, or relative motion of atomic layers, aided by motions of dislocations in the crystal grains as described later.

Permanent deformation is due to the breaking of bonds and relative motions of atomic layers in contrast to bond stretching in the elastic region. We should note that in the plastic region the bonds are still stretched and when the specimen is unloaded (say at point $P$) these stretched bonds uncoil. It is this uncoiling of stretched bonds that returns the stress–strain behavior along a line $(PO')$ almost parallel to the elastic deformation line.

As the load is increased, the specimen is plastically deformed further and eventually a point $M$ is reached when there appears to be less stress needed for further plastic deformation. At this point $M$, the specimen develops a small constriction or a neck at one location. The actual stress in the neck region is greater than anywhere else and all further deformation henceforth takes place in the neck which gets thinner and longer. Eventually the neck fractures at point $F$. From $Y$ to $M$ the plastic deformation is uniform throughout the central region of the specimen rod but from $M$ to $F$ it occurs locally in the neck region where the stresses are greatest due to the small cross sectional area of the neck.

As soon as the specimen fractures at $F$ the stretched bonds recoil which corresponds to the recovery of elastic deformation along the line $FB$. If we were to bring and fit together the two fracture pieces we would find that the gauge marks have moved apart from their original separation $L_o$ to $L_f$, which is the separation at fracture. The plastic fracture strain, $\varepsilon_{pl}$, or the strain after fracture and after elastic recovery, is the permanent deformation $OB$ in Figure 2-3. The stress at fracture, $\sigma_f$, is that stress at the instant of fracture $F$. If the load at instant of fracture is $F_f$ then $\sigma_f = F_f/A_f$. After fracture, the two broken pieces can be refitted together to obtain the cross sectional area at fracture $A_f$ (at the neck) by measuring the diameter $D_f$ at the neck. The true stress $\sigma_f$ at fracture is then $F_f/A_f$ and is more than $\sigma_f$, the apparent engineering stress.

It seems unusual that there appears to be less stress needed to deform the material from point $M$ to $F$. This is only a reflection of the less load needed from $M$ to $F$ not the actual stress in the specimen. The true stress, force per unit instantaneous area, actually increases because the cross sectional area of the neck decreases with deformation. If we were to plot the true stress vs. true strain behavior we would find the dashed curve shown in Figure 2-3. The true stress–strain behavior increases monotonically and does not
exhibit a maximum point \( M \). We always need to increase the true stress to further deform the sample. The points \( M' \) and \( F' \) on the true stress–strain curve correspond to \( M \) and \( F \). We should note that the relationship \( \epsilon_t = \ln(1+\epsilon) \) assumes uniform straining of the specimen and therefore does not apply beyond point \( M \) because from \( M \) to \( F \) extension is localized to the neck region. The true strain from \( M \) to \( F \) is larger than the engineering strain as sketched in Figure 2-3.

It is apparent from Figure 2-3 that once point \( M \) is reached necking occurs and the specimen proceeds to fracture in this neck region with less and less load. Point \( M \) therefore represents the maximum tensile stress that can be applied to the specimen without fracturing it. It is called the ultimate tensile strength or just \( \sigma_{TS} \) or \( TS \) of the specimen and it is an important engineering design quantity. The yield point \( Y \) that defines the elastic limit or the onset of yielding and plastic deformation defines the yield strength of the material, \( \sigma_y \) or \( YS \), which is another important design parameter. Yield strength represents the resistance of the material against plastic deformation. Higher is the yield strength, greater is the stress that can be applied without permanently deforming the specimen. Although in some materials the yield point \( Y \) is reasonably well defined in some other materials the transition from elastic to plastic deformation does not occur at such a well defined point.

Moreover, there is some uncertainty involved in locating the true yield point \( Y \) from elastic to plastic deformation regimes. It is therefore convenient to adopt a standard definition for yield strength that can be applied across the board for many materials. This is the 0.2% offset (or 0.2% proof) yield strength, \( \sigma_y(0.2\%) \). We take a strain 0.002 (0.2%) along the strain axis and draw a line parallel to the elastic deformation line as shown in Figure 2-3. This line cuts the stress–strain curve at point \( Y' \) which defines the offset yield strength\(^2\). According to Figure 2-3, the plastic strain at fracture, \( \epsilon_f \), represents the extent of plastic deformation a material exhibits before fracturing.

The greater is \( \epsilon_f \), the more ductile is the material. The ductility is defined as the amount of plastic deformation that is exhibited by a material at fracture. There are two quantitative definitions based on percent specimen elongation and percent cross sectional area reduction.

**Percent elongation** is plastic strain at fracture, \( \epsilon_f \), expressed as percentage,

\[
EL\% = \frac{L_f - L_o}{L_o} \times 100\% = \epsilon_f \times 100\%
\]

Percent elongation (2-1)

If \( A_o \) is the cross sectional area at fracture then \( A_o - A_f \) is the reduction in the cross sectional area. Percent area reduction \( AR\% \), is defined by

\[
AR\% = \frac{A_o - A_f}{A_o} \times 100\%
\]

Percent area reduction (2-2)

For a given material \( EL\% \) and \( AR\% \) values are different. Further, since most of the plastic elongation occurs in the small necked region rather than throughout the whole gauge length, \( L_o \), \( EL\% \) is higher for shorter \( L_o \) which must therefore be stated in comparing \( EL\% \) values between materials (for standard specimens, \( L_o = 2 \text{ in.} \)). \( AR\% \), on the other hand, does not depend on \( L_o \) or \( A_o \) and is therefore more frequently used to express ductility.

Ductility is obviously and important design parameter because we have to know the maximum amount of deformation a component can experience before fracture. In shaping metals, as in forging, ductility defines the limit of maximum allowable plastic deformation.

We can differentiate between ductile and brittle materials by comparing their stress–strain characteristics and their percent elongations or plastic strains at fracture (\( \epsilon_f \)) as shown in Figure 2-4. A ductile material exhibits an ultimate tensile strength point \( M \) and considerable plastic deformation before

\(^2\) In the UK, the offset is sometimes taken as 0.1% and the corresponding \( \sigma_{y(0.1\%)} \) strength.
fracture at point $F$. It has typical values of $EL\%$ in excess of 10%. Some materials, for example some cast irons and some metal alloys, exhibit only moderate ductility.

![Stress-Strain Curve]

Although it is possible to define a yield strength, the fracture occurs either before or around the ultimate tensile strength point $M$. Brittle materials do not exhibit any marked plastic deformation and their fracture strains are less than a few percent. Many ceramics, glasses and some metals, such as gray cast irons, are brittle materials. The fracture occurs suddenly which may be on the elastic region. Brittle fracture invariably involves induced crack propagation across the sample at sufficiently large stresses and the position of the fracture point, $F_{brittle}$, usually depends on the surface conditions of the specimen e.g. the presence of surface flaws, surface cracks etc.

For example, in a tensile test on a brittle material, it would not be unusual to have fracture by induced crack propagation across the specimen originating from one of the indentations of the extensometer grip pins. For brittle materials the tensile strength is usually taken as the fracture stress $\sigma_f$ at $F_{brittle}$ but its value is highly variable from sample to sample depending on various imperfections and flaws not only on the surface but also in the bulk of the brittle material.

We will see later that failure in brittle materials invariably involves crack propagation across the specimen. The fracture starts from a tiny, invisible, crack on the surface.

3 Elastic and Plastic Work. Resilience and Toughness

As the specimen is stretched work is done by the load. Initially this work is elastic work and is stored in the stretched bonds in the solid. When the load is removed, the work is returned by the specimen. **Resilience** of a material describes the extent of elastic energy stored per unit volume in the solid.

A material is said to be more resilient if it can store more elastic energy per unit volume of the specimen. We can find the maximum of elastic energy that can be stored per unit volume by evaluating the work done up to the yield point, end of elastic deformation behavior.

If $F$ is the load when the specimen is stretched by a small amount $dL$ then work done, $dW$, by the load is $dW = F \, dL$ and work done per unit volume $dW_{vol}$ is
\[ dW_{\text{vol}} = \frac{FdL}{A_oL_o} = \sigma d\varepsilon \]

Total work done per unit volume from zero strain to some strain \( \varepsilon \) is

\[
W_{\text{vol}} = \int_0^{\varepsilon} \sigma d\varepsilon
\]

Equation (3-1) shows that the work done per unit volume by the load bringing about a strain \( \varepsilon \) is simply the area under the stress-strain curve up to that strain. Modulus of resilience or simply resilience is the maximum elastic energy stored per unit volume which is the area under the \( \sigma - \varepsilon \) curve up to the yield point \( Y \) as illustrated in Figure 3-1 (a).

In this elastic region of the \( \sigma - \varepsilon \) characteristic the relationship between \( \sigma \) and \( \varepsilon \) is linear, \( \sigma = E\varepsilon \). Then, integrating Equation (3-1) with \( \sigma = E\varepsilon \) up to the yield strength point, the maximum elastic energy stored per unit volume in the stretched bonds is expressed by

\[
\text{Modulus of Resilience} = \frac{1}{2} E\varepsilon_y^2 = \frac{1}{2} \sigma_y\varepsilon_y = \frac{1}{2} \frac{\sigma_y^2}{E} \tag{3-2}
\]

Those solids with higher yield strengths and lower Young's moduli therefore possess greater resilience and are used in such applications as springs where it is important to store as much elastic energy as possible. During plastic deformation, work done by the load causes material flow and changes the shape of the specimen permanently. This work is not recoverable when the load is removed at any time and is called plastic work.

Total amount of plastic work done per unit volume in bringing the specimen to fracture is the area under the \( \sigma - \varepsilon \) curve, \( OMFBO \) in Figure 3-1 (b), which excludes the elastic portion, \( FBB'F \), that is returned when the specimen fractures and the stretched bonds spring back. This total amount of plastic work done per unit volume to fracture is called toughness and represents the energy absorbed by per unit volume of the specimen before it fractures.

Figure 3-1 (b) illustrates the definition of toughness. The more work needs to be done to fracture the specimen, the tougher it is. Toughness is therefore a useful comparative measure of the amount of
An energy that is expanded per unit volume in bringing about fracture. Examination of different \( \sigma - \varepsilon \) curves in Figure 2-4 shows that ductile materials tend to have greater toughness whereas brittle materials exhibit very limited toughness. Figure 3-2 shows four materials with varying ductility and strengths. Although both \( A \) and \( B \) have the same ductility, \( B \) is tougher than \( A \). Indeed \( B \) is the toughest amongst the four and \( D \), the brittle material, is the least tough. Although \( B \) has lower strength than \( C \), \( B \), nonetheless, is tougher than \( C \). \( B \) has higher strength than \( A \) but both have about the same toughness.

![Stress-strain graph](image)

**Figure 3-2**

Table 3-1 summarizes some of the typical mechanical parameters that may be obtained from a tensile test. Table 3-2 lists the yield and tensile strengths and ductility of a selection of materials. Ceramics and glasses are inherently brittle though they can possess high strengths. They fracture suddenly when the applied stress is able to propagate a crack-type defect across the specimen. Small crack like defects (normally invisible to the naked eye) can readily arise as surface imperfections. Their role in brittle fracture is discussed in a separate chapter. The tensile strengths of ceramics therefore tend to be variable depending on the conditions of the specimen.

For example, tensile strength of silicon nitride, Si\(_3\)N\(_4\), hot pressed with a porosity less than 1% is quoted as 350–580 Mpa. When reaction bonded with a 15–25% porosity, this is 100–200 MPa. In the form of a single crystal whisker (a very thin crystal) in which the surface is free of flaws, the tensile strength is some 1,400 MPa. Elastic deformation is said to occur *instantaneously* when a stress is applied to a body. The bonds between atoms become stretched or compressed almost instantaneously\(^3\) and the induced strain occurs immediately. Furthermore, when the stress is removed, the solid returns to its original shape and size.

---

\(^3\) Elastic deformations travel at the speed of sound so the time it takes to respond to a mechanical stress is the time it takes for the sound to travel down the solid, typically in the range microseconds to milliseconds.

\(^4\) 1kgf is the weight of 1kg, that is, approximately 10 N.
Table 3-1
Typical mechanical properties obtainable from a tensile test (tensile stress–strain characteristics)

<table>
<thead>
<tr>
<th>Mechanical Property</th>
<th>Symbol</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Young's (Elastic) Modulus</td>
<td>$E$</td>
<td>Slope of the $\sigma$–$\varepsilon$ curve in the initial linear (elastic) region or near the origin</td>
</tr>
<tr>
<td>Yield Strength</td>
<td>$\sigma_y$</td>
<td>Stress value ($F/A_o$) where elastic to plastic deformation occurs. Material begins to deform permanently when the stress exceeds $\sigma_y$</td>
</tr>
<tr>
<td>Offset Yield Strength $\sigma_y(0.2%)$</td>
<td></td>
<td>Used to characterize the yield strength of a material that yields gradually. Point where the line drawn from $\varepsilon = 0.002$ (on the strain axis) and parallel to the elastic line at the origin cuts the $\sigma = \varepsilon$ curve.</td>
</tr>
<tr>
<td>Tensile Strength</td>
<td>$\sigma_{TS}$</td>
<td>Stress value ($F/A_o$) where the $\sigma$–$\varepsilon$ curve is maximum and necking begins.</td>
</tr>
<tr>
<td>Stress at Fracture</td>
<td>$\sigma_f$</td>
<td>Stress value ($F_f/A_f$) where fracture occurs when the load is $F_f$.</td>
</tr>
<tr>
<td>Plastic Fracture Strain</td>
<td>$\varepsilon_f$</td>
<td>Plastic strain after fracture. The broken pieces are refitted and the new gauge length $L_f$ and hence plastic extension, $L_f - L_o$ are measured. $\varepsilon_f = (L_f - L_o)/L_o$. It represents the ductility of the material.</td>
</tr>
<tr>
<td>True Stress at Fracture</td>
<td>$\sigma_f$</td>
<td>True stress value ($F_f/A_f$) where fracture occurs. The broken pieces are refitted and the diameter and hence cross sectional area $A_f$ are measured in the narrowest neck region.</td>
</tr>
<tr>
<td>Percent Elongation</td>
<td>$EL%$</td>
<td>A measure of ductility. $100 \times \varepsilon_f$. Depends on $L_o$.</td>
</tr>
<tr>
<td>Resilience</td>
<td>$AR%$</td>
<td>A measure of ductility. If $A_f$ is the cross sectional area of the narrowest neck region after fracture.</td>
</tr>
<tr>
<td>Resilience Modulus of Resilience</td>
<td></td>
<td>The amount of elastic energy stored per unit volume of the solid due to elastic straining. Area under the $\sigma$–$\varepsilon$ curve up to the yield point.</td>
</tr>
<tr>
<td>Toughness Modulus of Toughness</td>
<td></td>
<td>The amount of work done in plastically deforming the specimen per unit volume to fracture. Area under the $\sigma$–$\varepsilon$ curve up to the fracture point but excluding the elastic energy stored.</td>
</tr>
</tbody>
</table>

Metals are generally used as alloys because by alloying we can control the mechanical properties. Many pure metals possess low strength but can be strengthened by alloying. It is customary to classify metals into low, medium, high and ultrahigh strength based on the yield strength. This scheme, somewhat arbitrary, is summarized in Table 3-3 with some typical examples.
Table 3-2
Typical mechanical properties of various materials (exact values depend on sample purity, polycrystallinity, previous plastic work)

<table>
<thead>
<tr>
<th>Material</th>
<th>ρ (g cm⁻³)</th>
<th>E (GPa)</th>
<th>σ_y (MPa)</th>
<th>σ_TS (MPa)</th>
<th>ε_f (%)</th>
<th>Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Many Ceramics</td>
<td>Variable</td>
<td></td>
<td>1,400</td>
<td></td>
<td>0</td>
<td>Brittle. Fracture invariably by crack propagation across the specimen from surface flaws or defects. Single crystal whisker</td>
</tr>
<tr>
<td>Si₃N₄</td>
<td>3.2</td>
<td>380</td>
<td>-</td>
<td>1,400</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>Si₃N₄</td>
<td>3.2</td>
<td>304</td>
<td>-</td>
<td>350-580</td>
<td>0</td>
<td>Hot pressed and porosity less than 1%</td>
</tr>
<tr>
<td>Titanium</td>
<td>4.51</td>
<td>107</td>
<td>170</td>
<td>240</td>
<td>25-30</td>
<td></td>
</tr>
<tr>
<td>Magnesium</td>
<td>1.74</td>
<td>45</td>
<td>41</td>
<td>165</td>
<td>4-10</td>
<td>Light weight. Limited ductility.</td>
</tr>
<tr>
<td>Copper</td>
<td>8.94</td>
<td>120</td>
<td>130</td>
<td>220</td>
<td>50-70</td>
<td>Very ductile.</td>
</tr>
<tr>
<td>Brass</td>
<td>8.53</td>
<td>110</td>
<td>220</td>
<td>450</td>
<td>35</td>
<td></td>
</tr>
<tr>
<td>Nickel (99.9%)</td>
<td>8.90</td>
<td>210</td>
<td>100</td>
<td>400</td>
<td>40</td>
<td>Annealed</td>
</tr>
<tr>
<td>Iron</td>
<td>7.87</td>
<td>200</td>
<td>100</td>
<td>270</td>
<td>45</td>
<td></td>
</tr>
<tr>
<td>1010 Steel (Low carbon)</td>
<td>7.86</td>
<td>207</td>
<td>180</td>
<td>325</td>
<td>30</td>
<td>Automobile panels, nails and wires</td>
</tr>
<tr>
<td>Gray cast iron</td>
<td>7.15</td>
<td>180</td>
<td>-</td>
<td>150</td>
<td>-</td>
<td>Brittle.</td>
</tr>
<tr>
<td>Ductile Cast Iron</td>
<td>7.12</td>
<td>165</td>
<td>270</td>
<td>400</td>
<td>18</td>
<td>Nodular ferritic cast iron</td>
</tr>
<tr>
<td>Aluminum</td>
<td>2.7</td>
<td>70</td>
<td>30</td>
<td>70</td>
<td>45</td>
<td></td>
</tr>
<tr>
<td>PMMA (plastic)</td>
<td>1.18</td>
<td>3</td>
<td>-</td>
<td>48-76</td>
<td>2-10</td>
<td></td>
</tr>
<tr>
<td>PET (Polyester)</td>
<td>1.29</td>
<td>3-4</td>
<td>-</td>
<td>48-72</td>
<td>30-300</td>
<td></td>
</tr>
</tbody>
</table>
Table 3-3
Strength classification of metal alloys

<table>
<thead>
<tr>
<th>Strength</th>
<th>$\sigma_y$ (MPa)</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>High strength</td>
<td>750–1,500</td>
<td>Tungsten. High strength low alloy (HSLA) steels, quenched and tempered steels. Beryllium-copper (97.9% Cu-1.9% Be-0.2% Co) alloy precipitation hardened. Titanium alloys.</td>
</tr>
</tbody>
</table>

Figure 3-3 shows typical stress vs. strain characteristics of various metals which exhibit ductility. Low carbon steels possess an upper yield point ($\sigma_{uy}$) at A and a lower yield point ($\sigma_{ly}$) at B as indicated in Figure 3-3.

Figure 3-3: Stress vs strain behavior for a few selected metals (data selectively extracted and combined from various sources) Steels (thick solid line) exhibit an upper yield point, marked A, and a lower yield point marked B.
Example 3.1: Plastic deformation and fracture of a steel wire

Consider a low carbon steel (1010) wire of diameter 5 mm. What is the maximum load it can carry before yielding? What is the maximum load for the wire to break?

Solution

We only need to determine the tensile loads for yielding and fracture. The yield and tensile strengths are given in Table 3-2.

For yielding, the load must be $F_y$,

$$F_y = A_o \sigma_y = \pi \left( \frac{D_o}{2} \right)^2 \sigma_y = \pi (2.5 \times 10^{-3} \text{ m})^2 (180 \times 10^6 \text{ Pa}) = 3.53 \times 10^3 \text{ N}$$

The load must be 3.53 kN and since $g \approx 10 \text{ ms}^{-2}$, this corresponds to attaching a mass of about 353 kg. The fracture occurs when the load reaches the tensile strength of the wire. If this force is $F_{max}$ then

$$F_{max} = A_o \sigma_{TS} = \pi \left( \frac{D_o}{2} \right)^2 \sigma_{TS} = \pi (2.5 \times 10^{-3} \text{ m})^2 (325 \times 10^6 \text{ Pa}) = 6.38 \times 10^3 \text{ N}$$

Thus, when the load reaches 6.38 kN, the wire fractures. This load corresponds to attaching a mass of about 638 kg.

Example 3.2: Comparison of mechanical properties

Consider the stress–strain characteristics of the metals shown in Figure 3-3. Identify the metals with the highest strength, greatest ductility, greatest toughness, lowest strength and lowest toughness?

Solution

Aluminum 7075-T6 has the highest strength. Copper is the most ductile. Brass has the greatest toughness. Copper has the lowest strength. Aluminum 2024-T81 probably has the lowest toughness.

4 Plastic Deformation in Metals and Dislocations

4.1 Dislocation Glide and Crystal Slip

The atomic origin of plastic deformation in metals involves the breaking of bonds and relative motions of layers of atoms. However, the layers of atoms do not all move together because this would require the breaking of all the bonds between this layer and the neighboring layers all at the same time which requires far too much energy. The actual plastic flow is the result of breaking and making bonds one line of atoms at a time through the motion of dislocations.

A crystal with an edge dislocation is shown in Figure 4-1 (a) where atom A is on the dislocation line running into the paper. The sign $\perp$ represent the edge dislocation and the position of the dislocation line where the vertical line is the extra half plane of atoms in the crystal. The atoms A, B, B', C, C', around the dislocation line are displaced from their normal equilibrium positions which means that bonds between them are already strained. For example, the bond $AB$ is shorter and bond $BC$ is longer than the equilibrium bond length.
(a) An edge dislocation in a crystal. (b) Under a sufficient shear strain (due to shear stress) $A$ and $C$ approach each other close enough to be able to form a bond. $C$ breaks its bond with $B$ and makes a bond with $A$. Thus $A$ and $C$ are bonded and $B$ is now dangling. (c) The effect is a shift in the dislocation by one atomic spacing. The dislocation has moved on the dislocation plane.

**Figure 4-1**

When a shear stress is applied to the crystal as shown in Figure 4-1 (b), the shear strain causes atoms $A$ and $B$ to be slightly displaced towards the right and atom $C$ to be slight displaced towards the left. Atom $A$ therefore gets closer to $C$ whereas atom $B$ moves away from $C$. The bond $BC$ becomes even more stretched. If the shear strain is sufficient to get $A$ close to $C$ and $B$ further away from $C$ then the stretched bond $BC$ breaks and a new bond $AC$ is formed. Consequently the dislocation becomes shifted to atom $B$ by one interatomic distance as depicted in Figure 4-1 (c). If the shear stress is continued to be applied then the dislocation motion repeats itself until this reaches the end of the crystal where it causes a step equal to an interatomic spacing as illustrated in the sequence of events (a) to (e) in Figure 4-2. The crystal is then said to have slipped. The motion of a dislocation under an applied shear stress is termed **dislocation glide**.

**Figure 4-2**

An applied shear stress to a crystal, if sufficiently large, can generate and glide a dislocation from one end of the crystal to the other as illustrated in Figure 4-2 (a) to (e). When a shear stress is applied to a crystal, as in (a), it can generate a dislocation at $A$ as in (b). As long as the shear stress is applied, this dislocation can glide from $A$ to $B$ and so on, as in (c) and (d). When eventually the dislocation line reaches the end of the crystal, it generates a step equal to the interatomic separation. Plastic deformation by dislocation gliding is analogous to the locomotion of a caterpillar which is depicted in the same series (a) to (e) in Figure 4-2. The caterpillar moves by lifting up one leg at a time. There is yet a further analogy. It is quite difficult to move a carpet on the floor by simply pulling it. It is possible, however, to shift it by first...
making a ruck at one end and then pushing this ruck to the end of the carpet somewhat similar to the way in which caterpillar's "ruck" (hump) moves forward.

Under shear stresses dislocation glide on the slip plane (dislocation glide plane) along a slip direction (dislocation glide direction). When a dislocation reaches the end of the crystal it causes a step, a permanent displacement, to appear. The crystal is said to have slipped. The crystal slips in a direction is perpendicular to the dislocation line.

**Figure 4-3**

It is apparent that edge dislocations play an important role in plastic deformation in metals. Figure 4-3 illustrates macroscopically, without the atomic details, how an applied shear stress to a crystal in (a) generates a dislocation in (b) which glides on a plane called the slip plane and in a direction called the slip direction. In (c) the dislocation reaches the end of the crystal and causes a *slip*, a permanent displacement. (d) is simply a different perspective of (c). The top and bottom portions of the crystal have been permanently displaced with respect to each other. The crystal has slipped. There is now a permanent shear strain as indicated by the dashed lines in (c).

*A slip plane* is an atomic plane on which a dislocation can glide. The step at the surface of a slipped crystal is said to be along a *slip line* because the dislocation is a line defect and when it reaches the surface it causes a step along a line as shown in Figure 4-3(d). The magnitude and direction of the slip, due to the motion of one dislocation entirely through the crystal as in Figure 4-3(d), is defined in terms of the displacement of the lower portion of the crystal with respect to the upper portion and is the *Burgers vector*, $\mathbf{b}$. It is apparent that for an edge dislocation induced slip $\mathbf{b}$ is an atomic spacing in the slip plane along the slip direction.

When shear forces are applied to the perfect crystal in Figure 4-2 (a), a dislocation is generated which is then moved from $A$ to $B$ and so on all the way to the end of the crystal. The crystal then becomes displaced by one atomic spacing as in Figure 4-2(e). Work is done by the shear forces in moving the dislocation across the crystal and slipping the crystal. Majority of this work, some 95% of it, generates lattice vibrations or heat while the dislocation is being "pushed" across the crystal. When we plastically deform a metal piece it gets warmer for this reason. When we repeatedly bend a piece of metal such as a metal clothes' hanger back and forward we can feel that the piece gets warmer.

*Screw dislocations* also contribute to plastic deformation, the only difference is in the direction of dislocation glide and crystal slip as illustrated in Figure 4-4 (a) and (b). For a screw dislocation the dislocation line and the Burgers vector, or slip direction, are parallel whereas the two are at right angles for an edge dislocation (Figure 4-3). This means that the slip plane is well defined for an edge dislocation, as indicated in Figure 4-3 (b), but there are a number of choices for the screw dislocation.
Any plane that contains the dislocation line and the Burgers vector in Figure 4-4 is possible. For example the screw dislocation in Figure 4-4 (c) starts gliding on plane \( A \) and then changes its glide direction and glides on plane \( B \). When it reaches the end of the crystal it causes a step with a slip line that is bent. Planes \( A \) and \( B \) are both slip planes because the slip direction, Burgers vector \( b \), lies in both planes. A screw dislocation can therefore cross-slip which means that it can change its glide from one plane to another that is parallel to Burgers vector, as in Figure 4-4 (c).

This is not the case for an edge dislocation. Screw dislocations have more choices for planes on which they can glide than edge dislocations and hence their contribution to plastic flow is more substantial.

Screw dislocations under shear stresses also give rise to plastic deformation. The crystal slip direction \( b \), however, is along the dislocation line. Screw dislocations can cross-slip from slip plane to another that is parallel to \( b \).

Figure 4-4

4.2 Slip Planes, Slip Directions and Critical Shear Stress \( \tau_{css} \)

Dislocation glide occurs preferentially on certain atomic planes, slip planes, and along certain directions, slip directions, in the crystal. The glide is easiest in a plane that is most densely packed. Intuitively this gives the least "bumpy glide". The glide is easiest in a direction that has the least atomic spacing or along the direction that is most densely packed. For example in Figure 4-1 (a) and (b), if \( A \) and \( B \) are closer then it is easier for the bond \( CB \) to switch to \( CA \).

Work done by the applied shear stress in moving the dislocation at \( A \) in Figure 4-1 (a) to \( B \) depends on the distance \( AB \). The shorter is \( AB \), the less work is needed. Further, bond switching from \( CB \) to \( CA \) in (b) is even easier if \( CB \) is longer than \( AB \). A long \( CB \) means a large interplanar separation. This is the case for densely packed planes which are separated by large interplanar distances.

Figure 4-5 (a) shows one of the four diagonal slip planes and the associated three possible slip directions in this plane in the FCC crystal structure. There are four distinct slip planes, diagonal planes, \( \{111\} \), in the crystal. Each has three slip directions of the type \( <110> \). There are therefore \( 4 \times 3 \) or 12 unique slip directions along which dislocation can glide with equal ease. All diagonal planes are slip planes and all \( <110> \) directions in these planes are slip directions as these are the most densely packed.

In the case of BCC crystals, as shown in Figure 4-5 (b), all 6 of the \( \{110\} \) type planes are the major slip planes. Each has 2 slip directions which are diagonal directions, \( <111> \) type, as shown in the Figure. There are therefore 12 major slip directions. There are other slip planes as well in the BCC structure that contain the \( <111> \) diagonal lines. If we were to count all these as well we would find 48 unique slip directions. In the HCP crystal only the basal plane, shown in Figure 4-5 (c), is the slip plane and this has three slip directions.

There are only 3 slip directions in the HCP crystal. HCP crystals, e.g. Cd, Mg, Zn, Ti, tend to be less ductile than FCC crystals which have 12 slip directions. All FCC metals possess high ductility as
exemplified by metals such as gold, copper, aluminum, nickel etc. BCC crystals have the same number of major slip directions as the FCC crystals but the crystal is not as densely packed. This means that BCC crystals require relatively greater shear stresses to move dislocations and hence appear to be stronger than FCC crystals.

![Slip planes and the associated slip directions in cubic crystals.](image)

**Figure 4-5**

There is a minimum shear stress that is required to move a dislocation on a slip plane along a slip direction in the perfect crystal as in Figure 4-3 and Figure 4-4. This minimum shear stress is a kind of intrinsic lattice friction stress which must be overcome to move the dislocation. It is called the *critical shear stress*, $\tau_{css}$ (formally known as the *Peierls-Nabarro stress*).

**Table 4-1**
Crystal Slip, Critical Shear Stress and Mechanical Properties

<table>
<thead>
<tr>
<th></th>
<th>FCC</th>
<th>BCC</th>
<th>HCP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Examples</td>
<td>Al, Ag, Au, Cu, Ni</td>
<td>Fe, W, Mo, Mn</td>
<td>Cd, Mg, Ti, Zn</td>
</tr>
<tr>
<td>Unique slip directions</td>
<td>12</td>
<td>48</td>
<td>3</td>
</tr>
<tr>
<td>Typical $\tau_{css}$ (MPa)</td>
<td>0.2 - 0.5</td>
<td>0.5 - 5</td>
<td>0.2 - 0.5</td>
</tr>
<tr>
<td>Typical relative mechanical property</td>
<td>Very ductile</td>
<td>Ductile</td>
<td>Limited ductility</td>
</tr>
<tr>
<td></td>
<td>Low yield strenght</td>
<td>High yield strength</td>
<td>Modest yield strength</td>
</tr>
</tbody>
</table>

Unless the applied shear stress $\tau$ exceeds $\tau_{css}$, a dislocation on a slip plane cannot be moved to yield plastic deformation. The critical shear stress, $\tau_{css}$, depends on the type of dislocation (edge, screw or mixed), atomic packing density and the nature of bonding in the crystal. Table 4-1 summarizes the possible slip directions and critical shear stresses for the three common metal crystal structures, FCC, BCC and HCP, and compares their mechanical properties.
4.3 Interactions of Dislocations and Pinning of Dislocations

The bonds in the crystal around a dislocation are strained by the presence of an additional half plane of atoms as depicted in Figure 4-6 (a). Above the dislocation line the bonds are in compression whereas below the line they are in tension. When two dislocations of the same sign (the additional half plane on the same side) are in the same vicinity then they will repel each other as shown in Figure 4-6 (a). If we were to try and move dislocation \( A \) close to \( B \) their tension zones would overlap and repel each other. Overlap of two tension zones would lead to a crystal lattice zone that is even more in tension. A similar argument also applies to the compression zones. Their overlap leads to a crystal zone even more in compression. Indeed when \( A \) and \( B \) are next to each other, there would be two additional half planes of atoms inserted and the associated crystal distortion, compression and tension, would be unacceptably large. It is apparent that the lattice strain energy increases as dislocations \( A \) and \( B \) are brought closer to each other and this corresponds to a repulsive force.

\[ \text{(a): Same sign dislocations repel each other.} \]

\[ \text{(b): Opposite sign dislocations attract each other.} \]

**Figure 4-6**

It can be seen that dislocation \( A \) cannot pass dislocation \( B \) and to two interfere with each other's motion. Two dislocations of opposite sign, shown as \( A \) and \( C \) in Figure 4-6 (b), attract each other. If we were to bring the dislocations \( A \) and \( C \) closer, the tension zone of \( A \) would overlap the compression zone of \( C \) resulting in a zone in which the bonds are not strained. The two zones effectively cancel each other. This reduction in the strain energy when \( A \) and \( C \) approach each other means that there must be an attractive force between the two dislocations. When \( A \) and \( C \) are brought together they form a continuous plane without any half planes of atoms which results in a perfect crystal. Thus two dislocations of opposite sign annihilate each other.

Plastic deformation depends on the motions of dislocations in the crystal but dislocation motion can easily be hindered by the presence of impurities and grain boundaries. This is illustrated in Figure 4-7. The dislocation is in grain 1. It can not simply glide right and pass the impurity atom when a shear stress is applied because the impurity atom has distorted, or strained, the crystal region around it.

In this example the bonds around the large substitutional impurity are compressed and if the dislocation tries to approach this region the bonds will have to be strained even more and this requires energy. Similar arguments also apply to small impurities around which the bonds are stretched. In both cases the crystal lattice around the impurity is strained and further straining by a dislocation requires too much energy. Impurities therefore act as obstacles against dislocation motion. When under a shear stress the dislocation moves towards the right the dislocation line around the impurity becomes pinned by the impurity. If the shear stress tries to move the dislocation in Figure 4-7 in grain 1 towards the left, then the dislocation meets a grain boundary.

This is a region of the crystal where there is no well-defined atomic order and bonds are already strained. There is a discontinuity in the crystal structure or crystal orientation from grain 1 to grain 2 and a marked change in the slip plane and direction. Thus it will take substantial energy (large shear stresses) to reorganize the passage of the dislocation from grain 1 to grain 2. Grain boundaries therefore hinder the glide of dislocations from one crystal grain into the other. Specimens of a given material that are more
polycrystalline, those that have more grains with smaller grain size, therefore possess higher yield strength as borne out by experiments.

The glide of an edge dislocation can be hindered or prevented by grain boundaries and point defects. The dislocation in the figure has to cross the grain boundary from the slip plane in grain 1 to the slip plane in grain 2 which is oriented in a different direction. The lattice distortion around the substitutional impurity prevents the dislocation from crossing this region.

Figure 4-7

4.4 Plastic Deformation of Polycrystalline Materials and Strain Hardening

Even under a purely tensile applied stress there are still shear stresses within a material and these shear stresses give rise to the observed plastic deformation. We consider a specimen of cross sectional area $A_o$ under a tensile stress $\sigma_o$ as shown in Figure 4-8.

The forces on the specimen are $F_o = \sigma_o A_o$. A thin layer $BC$ of the specimen inclined at an angle $\theta$ to the axis of the specimen along the force as shown in the figure. This layer has a surface area $A$ which is $A_o / \sin \theta$. Its top surface experiences a force $F_o \sin \theta$ normal to its surface and a tangential force $F_o \cos \theta$ acting on the surface itself.

Its bottom surface experiences a force $F_o \sin \theta$ normal to the surface and tangential force $F_o \cos \theta$ in the opposite direction to the tangential force on the upper surface. It is apparent that the two tangential forces on this layer are shearing forces and give rise to shear stresses, $\tau$, given by,

$$\tau = \frac{F_o \cos \theta}{A} = \frac{F_o \cos \theta}{A_o / \sin \theta} = \frac{F_o \cos \theta \sin \theta}{A_o}$$

Further, since $\sigma_o = F_o / A_o$ and $\sin(2\theta) = 2\sin \theta \cos \theta$, we can write the above as,

$$\tau = \frac{1}{2} \sigma_o \sin(2\theta)$$

Shear stress and applied stress

(4-1)

The shear stresses experienced by the layer $BC$ varies as $\sin 2\theta$ which means that it is maximum when the layer is inclined at $45^\circ$ as shown in the $\tau$ vs. $\theta$ plot in Figure 4-8. The significance of this is that those crystal slip planes that are inclined closest to $45^\circ$ will experience the largest shear stresses and will therefore participate in slip first. We have already mentioned that there is a minimum shear stress, called the critical shear stress, $\tau_{cs}$, that is required to move dislocations on the slip plane along the slip direction in a
perfect crystal. Unless the applied shear stress exceeds $\tau_{css}$, dislocations cannot be moved to cause plastic deformation.

An applied tensile stress gives rise to shear stresses acting on a layer of the material inclined at an angle $\theta$. The shear stresses are maximum when $\theta = 45^\circ$.

**Figure 4-8**

We will consider what happens when a polycrystalline specimen is put under a tensile load. Some of the grains will be already oriented to the tensile load axis in such a way that their slip planes will experience shear stresses in excess of the critical shear stress. However, these grains cannot by themselves slip and deform plastically because they are mechanically held by the neighboring grains. Indeed, all experiments suggest that when a polycrystalline material deforms, all the grains become plastically deformed in a similar fashion. For example they all become elongated as illustrated in Figure 4-9 (a) and (b). Initially, before the deformation, the grains were of random size and dimension but after plastic deformation they have all experienced permanent elongation along the direction of plastic extension. In other words, this means that the specimen does not come apart at the grain boundaries by the slip of favorably oriented grains. Thus plastic deformation in a polycrystalline material invariably involves all the
grains.

Schematic illustration of the grain structure of a polycrystalline material (a) before and (b) after plastic deformation. Before deformation all grains are of random size whereas after deformation they are all elongated in the same direction as specimen extension.

Figure 4-9

Plastic deformation does not begin until shear stresses in the majority of the grains are in excess of the critical shear stress Figure 4-10 shows the nature of plastic deformation a polycrystalline material. Dislocations glide in individual grain along planes inclined as close to 45° as possible. In some grains the slip plane will be inclined at an angle larger than 45° and in others it will be inclined at a lower angle than 45° but the average slip plane angle will be around 45°.

Plastic deformation in a polycrystalline material,

Figure 4-10

It may be thought that as long as the shear stresses exceed the critical shear stress in nearly all the grains, dislocations will glide and the material will continuously deform plastically. This would be idealized plastic deformation which is schematically shown as a broken line in the $\sigma$ vs. $\varepsilon$ characteristics in Figure 4-10. As soon as the applied stress reaches $\sigma_y$ at point Y, continuous plastic flow begins. However, as dislocations move around they become pinned by impurities and also become tangled up. An entangled dislocation cannot be moved easily. We are already aware of the dislocation–dislocation interactions and dislocation pinning mentioned above. Plastic flow does not continue unless we increase the applied stress.
To deform the material further from $Y$ to $A$ we have to apply more stress. For example to further deform the specimen permanently by $\varepsilon_a$ we need to increase the stress to $\sigma_a$. The material appears to harden. This is called strain hardening or work hardening and is due to the entanglement of dislocations in the material. It arises when we plastically strain the material. Figure 4-11 shows a photograph from a transmission electron microscope of a cold worked stainless steel foil. The dark entangled lines are the dislocations. An entangled dislocation cannot be glided by applied shear stress and therefore cannot contribute to plastic deformation.

Our personal experience on bending a metal clothes' hanger can readily confirm strain hardening. When you bend a clothes' hanger for the first time it is quite easy. When you bend it again, it gets a little harder. Each successive bending gets harder and eventually it fractures. Why does it get warmer as you bend it ?. Generally as the temperature increases dislocation motions become easier. For example a dislocation pinned by an impurity can overcome the potential energy barrier and pass the impurity as shown in Figure 4-7. Thermal fluctuations increase with temperature and hence also the probability of surmounting a potential energy barrier presented by an obstacle to the dislocation motion.

![Entangled dislocations in a cold worked stainless steel foil as seen by a transmission electron microscope, TEM (magnification more than 10,000). [from M. J. Whelan, P. B. Hirsch, R. W. Horne, and W. Bollmann, Proc. R. Soc. Lond. A 249]](image)

Both yield strength and tensile strength decrease with increasing temperature whereas the ductility increases as shown in Figure 4-12 for a titanium sheet. It is instructive to mention that although both tensile and shear stresses of adequate magnitude lead to plastic deformation, the application of a hydrostatic pressure to a solid does not lead to plastic deformation. When a body experiences a uniform hydrostatic pressure (obviously over all its surface), its volume get smaller by the compression of bonds. This elastic deformation takes up any increase in the hydrostatic pressure.

5 Cold Work and Recrystallization

We consider what happens to the mechanical properties of an annealed ductile metal as it is plastically deformed. Suppose that during a tensile test on this ductile metal we pass the yield point $Y$ as shown in Figure 5-1 and then at some point at $A$, before $M$, in the plastic deformation region we unload it. The specimen then recovers along the line $AO'$, parallel to the elastic region $OY$.

(a) A ductile metal specimen is taken to point A in its tensile stress-strain characteristic and then unloaded. The specimen returns along the elastic line $AO'$ and has permanent deformation equal to $OO'$. It has been cold worked. (b) If now we reapply the load, the specimen initially behaves elastically and does not yield until the stress is $A$. This is now the new yield strength, $\sigma_{y2}$, of the cold worked specimen which is greater than $\sigma_{y1}$ of the original specimen (c) The cold worked specimen has higher yield and tensile strengths but poorer ductility.

Figure 5-1
The recovery right after fracture corresponds to the springing back of the stretched atomic bonds. The specimen now has a permanent deformation $OO'$. It is longer and narrower. We say that the specimen has been cold worked; cold with reference to the melting temperature whose significance in this case will be discussed later in this section. If we were to reload this cold worked specimen as in Figure 5-1(b) we would find that it deforms elastically along $OA$ until point $A$, the point at which it was unloaded in (a). Its yield strength $\sigma_{y2}$ is therefore greater than the original yield strength, $\sigma_{y1}$, of the annealed specimen. The specimen is therefore stronger. If we were to carry out the tensile test to fracture, as shown in Figure 5-1(b), we would find that the tensile strength is also greater; $M'$ is higher in (b) than $M$ in (a). The two stress–strain characteristics, that of the annealed specimen and that of the cold worked specimen, are shown together in Figure 5-1(c). It is apparent that cold worked specimen has poor ductility as it fractures at $\varepsilon_f2 < \varepsilon_f1$. We can therefore summarize the observations as follows.

Cold work increases the yield and tensile strengths but decreases the ductility. This is in general true. These effects result mainly from entanglements of dislocations which then lose their ease of motion. Many metals are shaped or formed for a particular application by plastic deformation. Nearly all metal shaping operations make use of plastic deformation as illustrated in the examples in Figure 5-2(a) to (d). These are (a) rolling, (b) drawing, (c) extrusion and (d) forging and are almost self-explanatory in Figure 5-2. In each case the shaped metal component has been put through cold work as a result of plastic deformation. The mechanical properties of the shaped component are different than those of the starting metal. It is important therefore to quantify the amount of cold work done on a specimen to characterize the changes in the mechanical properties as a function of this cold work. Figure 5-3 shows a metal specimen of original cross sectional area $A_o$ and length $L_o$. As a result of plastic deformation its cross sectional area and length become $A$ and $L$ respectively. Cold work is defined as the percentage change in the cross sectional area as a result of plastic deformation,

$$\text{Cold work} = \frac{A_o - A}{A_o} \times 100\%$$

Metal forming or shaping operations are based on plastic deformation involve a degree of cold work.

Figure 5-2
Although the example in Figure 5-3 is for a cylindrical specimen, the above definition applies equally to other shapes that have a uniform cross sectional area along the specimen (that which does not change along the sample length).

\[
\text{Volume} = A \cdot L = A_o \cdot L_o = \text{Constant}
\]

<table>
<thead>
<tr>
<th>Length, (L)</th>
<th>Length, (L_o)</th>
</tr>
</thead>
<tbody>
<tr>
<td>COLD WORK</td>
<td></td>
</tr>
<tr>
<td>Area, (A_o)</td>
<td>Area, (A)</td>
</tr>
</tbody>
</table>

\[
\text{CW} = \left(\frac{A - A_o}{A_o}\right) \times 100
\]

Definition of cold work in terms of the change in the cross sectional area.

**Figure 5-3**

It is important to note that the volume of the specimen after plastic deformation remains the same. Plastic deformation involves crystal slips but the crystal structure does not change. This means that plastic deformation does not change the density of the material. Since the total mass and the density remain the same, the volume also stays the same. Hence plastic deformation is expressed by

\[
\text{Volume} = A_o \cdot L_o = A \cdot L
\]

Plastic deformation (5-2)

It is important to remark that the volume does not remain constant during elastic deformation unless the Poisson ratio is \(\frac{1}{2}\). Many materials data books give the mechanical properties of metals such as the yield and tensile strengths and ductility as a function of cold work. The general rule in all cases is an increase in the yield and tensile strengths and a reduction of ductility as the specimen is cold worked as shown in Figure 5-4 for four metals, two FCC metals, copper and brass, and two BCC metals, iron and 1040 steel, or Fe+0.4%C. Cold work results in the reduction of ductility. If a cold worked component is to be further shaped or formed it needs to be ductile again. For example the shaping processes in Figure 5-2 frequently involve several steps. It is not possible without fracture to simply roll out a thin sheet of metal by rolling in a very thick plate of the metal in Figure 5-2 (a). The process needs to be done in several steps with each step requiring a high degree of ductility in the starting material. To restore the ductility of a cold worked specimen it is necessary to anneal the sample at a high temperature for a certain time called the annealing time. During annealing, the whole material is recrystallized whereby new dislocation free crystal grains first nucleate and grow as illustrated in the sequence in Figure 5-5 where a drawn rod is annealed at a high temperature to restore ductility. A cold worked specimen, as shown in Figure 5-5, has a microstructure in which all the grains have been strained and therefore contain a large number of tangled dislocations which is, in fact, the reason for strain hardening (the surfaces of the grains have numerous slip lines which can be easily seen under an optical microscope). A large dislocation density means that the crystal is in a high energy state. Its potential energy is greater than that of the dislocation free crystal which has minimum potential energy. If the specimen is subsequently annealed at a sufficiently high temperature then atomic diffusion becomes sufficiently rapid to allow some of the atoms around grain boundaries and some entangled dislocation regions to diffuse and form nuclei of strain–free (dislocation–free) crystals that have a nearly perfect crystal structure. Once new grains have been nucleated they grow and eventually encompass the whole material. This process is called recrystallization. The recrystallized specimen is highly polycrystalline contains many grains of various sizes. If the annealing is continued after recrystallization then large grains grow at the expense of small grains and the polycrystalline structure becomes coarser. This process is called grain growth or grain coarsening.
Changes in the yield strength, tensile strength and ductility as a function of cold work in FCC, (a) Copper, (b) Brass and BCC metals, (c) Iron and (d) 1040 Steel (Fe-0.4%C).

**Figure 5-4**

The larger the grain size, the smaller is the grain boundary area per unit volume of the material and the more ductile the specimen because grain boundaries hinder dislocation motions. Sometime after recrystallization, the specimen is brought down to room temperature. The specimen now has its ductility restored but has lost its strength gained from strain hardening. The microstructure of the specimen and the evolution of the yield strength and ductility during annealing are schematically shown in Figure 5-5. As the recrystallization process involves atomic diffusion, the rate depends on the diffusion coefficient and therefore it is thermally activated. The rate of recrystallization, $R$, is given by

$$R = R_0 \exp\left(-\frac{\Delta H_{\text{recryst}}}{RT}\right)$$

where $R_0$ is a constant (the rate at $T = \infty$) and $\Delta H_{\text{recryst}}$ is the activation energy for the recrystallization process (Joules per mole), $R$ is the gas constant and $T$ is the absolute temperature (always
The recrystallization rate is virtually zero at temperatures well below about one third of the melting temperature. It becomes significant when temperature is between \( \frac{1}{3}T_m \) to \( \frac{1}{2}T_m \) where \( T_m \) is the melting temperature. The recrystallization rate also depends on the extent of cold work although this is not directly apparent in Equation (5-3); it affects \( R_o \) and \( \Delta H_{\text{recryst}} \).

The time required for the recrystallization process is inversely proportional to the recrystallization rate and decreases dramatically with increasing temperature because of the exponential dependence in Equation (5-3). It is useful to have some idea of the temperature where the recrystallization process is significant. Recrystallization temperature, \( T_{\text{recryst}} \) is that temperature where the recrystallization process is complete in 1 hour. Typically the recrystallization temperature lies between \( \frac{1}{3}T_m \) and \( \frac{1}{2}T_m \) and is often taken to be around 0.47\( T_m \). Table 5-1 lists the recrystallization temperatures for some pure metals and alloys along with their melting temperatures.

Figure 5-5
Table 5-1
Typical Recrystallization temperatures (recrystallization complete in 1 hour). Note that the recrystallization
temperature also depends on the amount of cold work.

<table>
<thead>
<tr>
<th>Metal</th>
<th>Melting temperatures</th>
<th>Recrystallization Temperatures</th>
<th>( T_{\text{recryst}}/T_m ) and Comment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lead</td>
<td>232° C (505 K)</td>
<td>-4° C (269 K)</td>
<td>0.53. Cannot be cold worked. Recrystallizes almost immediately at room temperature.</td>
</tr>
<tr>
<td>Tin</td>
<td>327° C (600 K)</td>
<td>-4° C (269 K)</td>
<td>0.45. Cannot be cold worked. Recrystallizes almost immediately at room temperature.</td>
</tr>
<tr>
<td>Zinc</td>
<td>420° C (693 K)</td>
<td>10° C (283 K)</td>
<td>0.40. Cannot be cold worked.</td>
</tr>
<tr>
<td>Aluminum (Pure)</td>
<td>660° C (933 K)</td>
<td>80° C (353 K)</td>
<td>0.38. Cannot be cold worked</td>
</tr>
<tr>
<td>Aluminum</td>
<td>660° C (933 K)</td>
<td>300° C (573 K)</td>
<td>0.61. Aluminum that has been 75% cold worked.</td>
</tr>
<tr>
<td>Copper (Pure)</td>
<td>1085° C (1358 K)</td>
<td>121° C (394 K)</td>
<td>0.29. Pure Cu (99.999%). Cold rolled 98%.</td>
</tr>
<tr>
<td>Iron</td>
<td>1538° C (1811 K)</td>
<td>450° C (723 K)</td>
<td>0.40</td>
</tr>
</tbody>
</table>

Data selectivity extracted and combined from various sources.

It can be seen that for certain pure metals such as tin, lead and zinc the recrystallization temperature is below room temperature. Indeed for tin and lead the temperature is -4 °C. At this temperature the recrystallization takes about 1 hr but at room temperature it is orders of magnitude shorter, perhaps in the milliseconds range. This means that as soon as lead and tin are plastically deformed they immediately recrystallize and restore their ductility and therefore cannot be cold worked; they are always ductile at room temperature and cannot be strain hardened. It is apparent that when plastic deformation is carried out at a temperature above the recrystallization temperature then recrystallization occurs almost instantaneously and strain hardening cannot take place as in the case of tine and lead. This makes it much easier to plastically deform and shape metals without fracture at temperatures above the recrystallization temperature. For example iron above 450 °C can be plastically deformed by any amount without fracture (as well known by blacksmiths). Metal shaping processes above the recrystallization temperature, typically above \((0.5–0.6)T_m\), are called hot-working.

6 Mechanical Strengthening Mechanisms

It is apparent that plastic deformation depends on the ease with which dislocations can move in the material. Any process that reduces dislocation mobility increases the strength.

**Strain Hardening (Cold Work):** As mentioned before cold work leads to the generation of dislocations which become entangled and hinder further dislocation motions. Consequently the material exhibits higher yield strength as it is plastically deformed. This type of increase in the strength as a result of permanent strain is called strain hardening. Strain hardening reduces the ductility of the metal. Some metals such as lead, tin and zinc, cannot be strain hardened as their recrystallization temperatures are below the room temperature.

**Grain Size Hardening:** Grain boundaries are good obstacles against the motion of dislocations because a dislocation trying to cross from one slip plane in a given grain to another slip plane in a neighboring grain needs considerable reorganization of the dislocated atoms as depicted in Figure 4-7. It takes too much energy to push a dislocation through a grain boundary which is intuitively apparent in Figure 4-7. The presence of grain boundaries therefore increases the critical stress required to move a dislocation through the material for slip. The more grain boundaries we have, the greater is the yield. The presence of grain boundaries therefore increases the critical stress required to move a dislocation through the material for slip. The more grain boundaries we have, the greater is the yield strength.
There is a convenient relationship between the yield strength and the average grain size called the Hall-Petch equation. This is given by

$$\sigma_y = \sigma_{yo} + Kd^{-1/2}$$  \hspace{1cm} \textit{Hall-Petch equation} \hspace{1cm} (6-1)

where $d$ is the average grain size (diameter), $\sigma_{yo}$ and $K$ are metal-dependent constants. Grain size can be controlled by the solidification process as well as heat treatment. For example, annealing at elevated temperatures allows large grains to grow at the expense of small grains, increases the average grain size and hence lowers the strength. The ductility becomes larger. On the other hand, rapid solidification of a metal from the liquid frequently leads to smaller grains and hence to a higher yield strength.

**Impurity Hardening or Solid Solution Hardening:** Impurities, whether small interstitial atoms or substitutinal atoms, upset the perfect crystal lattice and give rise to strain fields as shown in Figure 4-7. They act as obstacles against dislocations and a dislocation around a impurity become pinned. It can be moved only by applying a higher stress than that for moving a dislocation in a perfect crystal. Thus the material exhibits a greater yield strength. Many alloys consist of atoms of one type of metal homogeneously mixed, or dissolved, into the crystal structure of another type of metal. For example, brass has Zn atoms substituting for Cu atoms and the alloy is Zn atoms dissolved the Cu FCC crystal. Zn atoms distort the crystal lattice and this distortion hinders dislocation motion. The alloy therefore possess greater yield strength.

**Dispersion or Precipitation Hardening:** Certain materials, as a result of heat treatment, can be made to have a microstructure that contains dispersed or precipitated particles within a matrix of a host crystal. For example, aluminum alloys such as Al-4% Cu can be heat treated so that their polycrystalline structure is aluminum rich (96% Al) FCC crystal grains which contain dispersed CuAl₂ crystals as schematically shown in Figure 6-1. In essence the copper atoms have formed dispersed compound crystals of CuAl₂ which are hard. These randomly dispersed particles in the Al grains act as obstacles against the motion of a dislocation and therefore strengthen the alloy.

The dispersed particles, here CuAl₂ crystals, have a distinctly different composition and crystal structure than the host which is 96% Al and FCC crystal grains. There is a sharp grain boundary between the two phases and a major change in atomic structure. The dislocation cannot therefore simply cross the boundary and pass through the CuAl₂ crystal without a substantial applied stress. The result is that the material possess a higher yield strength than it would have without the dispersed particles.

**7 Hardness**

Hardness of a material can be identified with its resistance to indentation, or penetration. It is determined by pressing an indenter into the surface of the material and measuring the size of the permanent impression. Obviously, the larger is the impression, the softer is the material. Thus the hardness of a material is a qualitative indication of its strength. Hardness tests are routinely used in industry to asses the
mechanical properties of a material quickly and inexpensively. If small loads are used, then the hardness test can be classified under non-destructive testing (NDT). Such small load indentation measurements are called microhardness tests.

7.1 Vickers Hardness

Vickers hardness test involves pressing a diamond indenter into the surface of the material at a constant load, $F$ (measured in kgf). The diamond indenter has the shape of a square pyramid with an apex angle $\theta$ of 136° as shown in Figure 7-1. After a fixed loading duration of say 10 seconds, the pyramidal indenter is removed and the diagonals, $d_1$ and $d_2$, of the permanent impression are measured. Vickers hardness number, VHN, is then simply load per unit surface area of the impression. If $d$ is the average diagonal in millimeters, then Vickers Hardness Number, VHN (kgf/mm$^2$), is given by

\[ VHN = \frac{F}{\text{Surface Area}} = \frac{F}{\frac{d^2}{2\sin\left(\frac{1}{2}\theta\right)}} = 1.854 \frac{F}{d^2} \]  

Vickers Hardness Number (7-1)

where $d = (d_1+d_2)/2$ is the average diagonal in millimeters and $\theta$ is the apex angle. Note that the conventional units for VHN are kgf/mm$^2$ so that to convert to MPa, the VHN in kgf/mm$^2$ must be multiplied by $g$ (≈ 10 ms$^{-2}$). As soon as the diamond tip starts penetrating the material, it plastically deforms the region around it which is shown as the gray region around the tip in Figure 7-1.

The instantaneous shear stresses around the indenter tip are given by the tangential force, $F \cos(\theta/2)$, divided by the instantaneous surface area of contact, $F \sin(68^\circ)/A$ or 0.37$F/A$. As the penetration and hence plastic deformation continues the contact area $A$ increases so that 0.37$F/A$ decreases. Penetration can only continue as long as the instantaneous shear stresses at the tip, as gauged by 0.37$F/A$, are greater than the yield strength of the material. Otherwise the penetration comes to a halt. 0.37$F/A$ when the diamond indenter has to come to a halt is then a measure of the yield strength of the material if the material were not to strain harden during the penetration.

The theoretical analysis of the stresses and plastic strains around the indenter tip involve stress mechanics, a topic far beyond the scope of this book. It is apparent, however, that only a constant proportion of $F/A$, is involved in plastic deformation and that the final value of $F/A$, when the indenter comes to a halt, gauges the yield strength of the material. It has been found that for many metals,

\[ VHN \approx 3\sigma_y \]  

Vicker hardness and yield strength relationship (7-2)

where VHN is in Pascals instead of kgf mm$^2$. Vickers test is one of the most widely used hardness measurements in research and development and provides a continuous range of VHN values. Table 7-1 summarizes some typical VHN values and the corresponding yield strengths of various metals.

<table>
<thead>
<tr>
<th>Material</th>
<th>Lead</th>
<th>PMMA</th>
<th>Al</th>
<th>Cu</th>
<th>Steel</th>
<th>Glass</th>
<th>Quartz</th>
<th>Tungsten</th>
<th>Carbide</th>
</tr>
</thead>
<tbody>
<tr>
<td>VHN (kgf/mm$^2$)</td>
<td>6</td>
<td>20</td>
<td>22</td>
<td>43</td>
<td>210</td>
<td>550</td>
<td>1200</td>
<td>2500</td>
<td></td>
</tr>
</tbody>
</table>

Table 7-1
Vickers hardness for various materials and correlation with yield strength for metals
7.2 Brinell Hardness

This hardness test was originally developed by the Swedish metallurgist Johan Brinell. A spherical indenter, such as a hard steel ball, is pressed into the surface of a material and the resulting diameter, \( d \), of the impression is optically measured as shown in Figure 7-2. The standard load for ferrous alloys is 1000 kg and it is applied for a duration of 30 seconds. For non-ferrous alloys the load is reduced to 500 kg. For very hard metals a tungsten carbide ball is used. The Brinell Hardness Number (BHN) is then simply the applied force per unit surface area of the impression, and is given by

\[
BHN = \frac{F}{\text{Surface Area}} = \frac{F}{\pi D^2 \left( D - \sqrt{D^2 - d^2} \right)}
\]

where \( F \) is the load (kgf), \( D \) is the diameter of the ball (mm) and \( D \) is the indentation diameter (mm). From the measurement of \( d \), BHN is normally read off Brinell hardness tables or calculated from Equation (7-3). The units for BHN are the same as those for VHN, kgf/mm\(^2\) and we need to multiply by \( g \), acceleration due to gravity, to convert to Pascals.
The Brinell hardness test is based on applying a large load, 1000 kg, and results in indentations which are large enough for surface irregularities to be neglected. Tables in various handbooks can be used to convert from one hardness scale to another. Conversion is dependent on the material tested.

7.3 Rockwell Hardness

Rockwell hardness test is widely used in North America, perhaps because of its flexibility for automation or, more likely, the originator was an American metallurgist, Stanley Rockwell. The indenter used for carrying out the Rockwell test can be either a hard sphere, for example a steel ball, or a conical diamond tip ("brale") as shown in Figure 7-3.

The Rockwell hardness test is carried out by first applying a minor load of 10 kg. A major load of 60, 100 or 150 kg is then applied and the increase in the indentation depth, \( t \), is recorded as shown in Table 7-2.

**Table 7-2**
Rockwell hardness scales

<table>
<thead>
<tr>
<th>Rockwell Test, Scale</th>
<th>Indenter</th>
<th>Load ( F ) (kg)</th>
<th>Formula ( R )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A )</td>
<td></td>
<td>60°</td>
<td></td>
</tr>
<tr>
<td>( C )</td>
<td>Diamond cone with an apex angle 120°</td>
<td>150°</td>
<td>( R = 100 - 500 \ t )</td>
</tr>
<tr>
<td>( D )</td>
<td></td>
<td>100°</td>
<td></td>
</tr>
<tr>
<td>( B )</td>
<td></td>
<td>100°</td>
<td></td>
</tr>
<tr>
<td>( F )</td>
<td>Steel ball with a diameter 1/16 in.</td>
<td>60°</td>
<td></td>
</tr>
<tr>
<td>( G )</td>
<td></td>
<td>150°</td>
<td>( R = 130 - 500 \ t )</td>
</tr>
<tr>
<td>( E )</td>
<td>Steel ball with a diameter 1/8 in.</td>
<td>100°</td>
<td></td>
</tr>
<tr>
<td>( H )</td>
<td></td>
<td>60°</td>
<td></td>
</tr>
</tbody>
</table>

The Rockwell hardness, \( R \), is then determined via \( R = C - 500 \ t \) where \( C \) depends whether the indenter was a diamond cone (\( C = 100 \)) or a steel ball (\( C = 130 \)). Thus the increase in the indentation depth, \( t \), is a measure of the Rockwell hardness of the material. Inasmuch as the Rockwell hardness is dependent on both the major load and the indenter, for each major load, and different indenter, there is a separate Rockwell hardness scale labeled by a capital letter subscript. Table 7-2 summarizes the different Rockwell hardness scales. There is some overlap between the various Rockwell hardness scales for \( R_B \) and \( R_C \). When the Rockwell hardness value for a measurement is below 20, it is best to use the next softer scale. For example \( R_C = 20 \) is about the same as \( R_B = 95 \).

A dial gauge which is used to measure the penetration depth increment, \( t \), on applying a major load is usually calibrated to read the Rockwell hardness directly which means that the test can be easily
automated (Rockwell hardness tester instruments are available commercially that numerically display the hardness). The hardness of many engineering metals are typically characterized by Rockwell hardness values in the literature, particularly in the USA. Tables and scales are available to convert from a Rockwell hardness value to other hardness scales but the conversion is material dependent.

7.4 Hardness and Strength

It is clear that the hardness value for a metal is an indication of its resistance to plastic deformation and should therefore be related to the yield strength of the material. In fact theoretical work shows that the yield strength \( \sigma_y \) of a metal and VHN are related as given in Equation (7-3), \( \text{VHN} \approx 3\sigma_y \). There are also correlations between the various hardness parameters and the tensile strength of metals. For example, for most steels, it has been found that the tensile strength, \( \sigma_{TS} \), can be correlated to the BHN via the empirical relationship,

\[
\sigma_{TS} (\text{MPa}) = 3.45 \times \text{BHN(kgf/mm}^2) \quad \text{Tensile strength and brinell hardness} \quad (7-4)
\]

Vickers hardness number has been similarly related to the tensile strength of steels which is not surprising as the definitions of BHN and VHN are indeed the same, load per unit area of impression. There are many graphs in materials data books that correlate the hardness of a metal with its yield and tensile strengths. It should be mentioned that although the correlation is material dependent, for a given class of materials, for example steels, there is typically an approximate linear relationship between the hardness and tensile strength.
**NOTATION**

\[ A \] cross sectional area (m^2)

\[ AR\% \] percent area reduction (also known as resilience)

\[ BHN \] Brinell Hardness Number (kgf/mm^2)

\[ CW\% \] cold work percentage

\[ D \] diameter (m)

\[ \Delta \] change

\[ \Delta \] volume strain (no units)

\[ \Delta H_{recryst} \] activation energy for the recrystallization process (J mol\(^{-1}\))

\[ \Delta V \] amount of change in a volume (m\(^3\))

\[ d \] average diagonal (mm)

\[ \delta \] a small change, an incremental change

\[ \delta \varepsilon \] incremental true strain (no units)

\[ dW \] work done

\[ E \] elastic modulus (Young’s modulus) (Pa)

\[ EL\% \] percent elongation (also know as ductility)

\[ \varepsilon \] engineering tensile strain (no units)

\[ \varepsilon_f \] plastic fracture strain (no units)

\[ \varepsilon \] true strain (no units)

\[ \varepsilon_x \] lateral strain (no units)

\[ F \] force (N); point of fracture

\[ f \text{ (subscript)} \] at fracture (e.g. \( L_f \) = separation at fracture)

\[ G \] shear modulus (also known as modulus of rigidity) (Pa or N m\(^{-2}\))

\[ g \] acceleration due to gravity (9.81 m s\(^{-2}\))

\[ h \] depth (m)

\[ K \] bulk modulus; compressibility (Pa or N m\(^{-2}\))

\[ \gamma \] shear strain (no units)

\[ L \] length (m)

\[ M \] ultimate tensile strength point

\[ o \text{ (subscript)} \] original (e.g. \( L_o \) = original length)

\[ P \] pressure (Pa)

\[ \theta \] shearing angle

\[ R \] rate of recrystallization; gas constant (\( N\_k = 8.31457 \text{ J mol}^{-1} \text{ K}^{-1} \); Rockwell hardness

\[ \rho \] density (kg m\(^{-3}\))

\[ \sigma \] engineering tensile stress (Pa or N m\(^{-2}\))

\[ \sigma_{TS} \] tensile strength (Pa or N m\(^{-2}\))

\[ \sigma_{y} \] yield strength (Pa or N m\(^{-2}\))

\[ \sigma_{y(0.2\%)} \] 0.2% offset yield strength (Pa or N m\(^{-2}\))

\[ T \] temperature (K)

\[ TS \] tensile strength

\[ T_{recryst} \] the temperature at which recrystallization takes one hour

\[ \tau \] shear stress (Pa or N m\(^{-2}\))

\[ \tau_{crt} \] critical shear stress (Pa or N m\(^{-2}\))

\[ \nu \] Poisson’s ratio

\[ VHN \] Vickers Hardness Number (kgf/mm\(^2\) or Pa, depending on which relationship is used)

\[ W_{vol} \] total work done per unit volume

\[ Y \] yield point

\[ YS \] yield strength

**USEFUL DEFINITIONS**

**Brittle** materials do not exhibit any marked plastic deformation and their fracture strains are less than a few percent. They exhibit very limited toughness. Ceramics and glasses are examples of brittle materials.

**Bulk Modulus** (\( K \)) is volume stress (pressure) needed per unit elastic volume strain, and is defined by \( p = -K\Delta \) where \( p \) is the applied volume stress (pressure) and \( \Delta \) is the volume strain. It gauges the extent to which a body can be reversibly (and hence elastically) deformed in volume by an applied pressure in terms of the material properties.

**Cold Work** is defined as the percentage change in the cross sectional area as a result of plastic deformation. A ductile metal can be cold worked by taking it to a point, \( A \), in the plastic deformation region in its tensile stress-strain characteristic, and then unloading it. The metal will return along a new elastic line, parallel to the old one. The point \( A \) is then the new yield strength of the metal. A cold worked material has higher yield and tensile strengths, but poorer ductility.

**Dislocation** is a line imperfection within a crystal that extends over many atomic distances.

**Ductile** materials exhibit an ultimate tensile strength point and considerable plastic deformation before fracture. They also tend to have a high toughness.
Edge dislocation is a line imperfection within a crystal which occurs when there is an additional but short plane of atoms which does not extend as much as its neighbors. The edge of the plane constitutes a line of atoms where the bonding is irregular and defines a line imperfection that is called an edge dislocation.

Elastic Deformation is temporary deformation that is caused by the application of a stress. When the stress is removed, the object returns to its original shape.

Elastic Modulus or Young's modulus ($E$) is axial stress needed per unit elastic axial strain, and is defined by $\sigma = E \varepsilon$ where $\sigma$ is the applied stress and $\varepsilon$ is the strain all along the same direction (axis). It gauges the extent to which a body can be reversibly (and hence elastically) deformed by an applied load in terms of the material properties.

Plastic Deformation is permanent deformation by material flow as a result of the motion of dislocations.

Poisson's ratio is the ratio of lateral to longitudinal strain for a particular material.

Resilience of a material is the amount of elastic energy it can store per unit volume of material.

Screw dislocation occurs when one portion of a perfect crystal is twisted or skewed with respect to another portion on one side of a line only.

Shear Modulus ($G$) is shear stress needed per unit elastic shear strain, and is defined by $\tau = G \gamma$ where $\tau$ is the applied shear stress and $\gamma$ is the shear strain. It gauges the extent to which a body can be reversibly (and hence elastically) deformed by shearing forces in terms of the material properties.

Strain is a measure of the deformation a material exhibits under an applied stress. It is expressed in normalized units. Under an applied tensile stress, strain ($\varepsilon$) is the change in the length per unit original length, $\Delta L / L_0$. When a shear stress is applied, the resulting deformation involves a shear angle. Shear strain is defined as the tangent of the shear angle that is developed by the application of the shearing force. Volume strain $\Delta$ is the change in the volume per unit original volume; $\Delta = \Delta V / V$.

Stress is force per unit area, $F / A$. When the applied force is perpendicular to the area it leads either to a tensile or compressive stress, $\sigma = F / A$. If the applied force is tangential to the area, then it leads to a shear stress, $\tau = F / A$.

Tensile Strength ($\sigma_{TS}$) is the maximum stress that can be applied to the specimen without fracturing it.

Toughness is the amount of plastic work done per unit volume to fracture.

Yield Strength is the resistance of the material against plastic deformation.

Young's modulus see Elastic Modulus

### QUESTIONS AND PROBLEMS

1. Consider a 5 meter steel wire of diameter 2.5 mm that is carrying a tensile load of 1 kN (equivalent to a mass of about 100 kg). If the elastic modulus and Poisson's ratio of steel are 210 GPa and 0.27 what is the new length and diameter of the steel wire assuming that the elongation is elastic? Compare the true and engineering strains and also true and engineering stresses.

   [Ans: New length = 5.0049 m; new diameter = 2.4993 mm; engineering strain = 0.09700%; true strain = 0.09696%; engineering stress = 203.72 MPa; true stress = 203.83 MPa]

2. An aluminum alloy (30003-H14) rod which is 10 mm in diameter and 1 m in length is subjected to a 6 kN tensile load. What is the new length and diameter of the rod given that the Young's modulus for this alloy is 70 GPa, yield strength is 145 MPa and the Poisson ratio $\nu = 0.33$. What is the elastic energy stored per unit volume?

   [Ans: New length = 1.0011 m; new diameter = 9.99640 mm; elastic energy = 41.7 kJ m$^{-3}$ or 0.0417 J cm$^{-3}$]

3. A tension test is carried out on an Al alloy specimen which has an original diameter of 0.505 in. After fracture, the diameter at the fractured neck is measured to be 0.37 in and the gauge length after fracture is 2.32 in (original gauge length was 2 in). The maximum load during the test was 37 whereas at fracture the load was 34 kN.

   a. What is the tensile strength?
b What are the engineering and true stresses at fracture?
c What are the engineering and true strain at fracture?
d What is the ductility of this metal?
[Ans: a 286 MPa; b 263 MPa, 490 MPa; c 16.0%, 14.8%; d EL% = 16%; AR% = 46%]

4. A tension test has been carried out on a steel specimen and the results shown in Table Q.4 have been obtained. The following information is available on the specimen: \(D_o\) = Original diameter = 12.67 mm; \(L_o\) = original gauge length = 50.80 mm; \(D_f\) = diameter at fracture = 9.00 mm; \(L_f\); gauge length at fracture = 59.31 mm; stress at fracture = 472.7 MPa; strain at fracture = 0.168. By suitable plots find the following.
a What is the elastic modulus?
b What is the 0.2% proof yield strength?
c What is the ultimate tensile strength?
d What is the ductility in EL% and AR%?
e What is the true stress at fracture?
f Estimate the toughness.
g Estimate the modulus of resilience.

Table Q.4 Tensile test results on a steel specimen. (F means fracture.)

<table>
<thead>
<tr>
<th>Strain</th>
<th>0.00025</th>
<th>0.00035</th>
<th>0.00055</th>
<th>0.00070</th>
<th>0.00090</th>
<th>0.00115</th>
<th>0.00130</th>
<th>0.00150</th>
<th>0.00170</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stress MPa</td>
<td>0</td>
<td>39.66</td>
<td>79.32</td>
<td>119</td>
<td>159</td>
<td>198</td>
<td>238</td>
<td>277.6</td>
<td>317</td>
</tr>
<tr>
<td>Strain</td>
<td>0.00190</td>
<td>0.00210</td>
<td>0.00230</td>
<td>0.00255</td>
<td>0.00300</td>
<td>0.00390</td>
<td>0.00510</td>
<td>0.00760</td>
<td>0.0126</td>
</tr>
<tr>
<td>Stress MPa</td>
<td>396.6</td>
<td>436</td>
<td>476</td>
<td>515.6</td>
<td>555</td>
<td>594.9</td>
<td>626.6</td>
<td>636.9</td>
<td>645.6</td>
</tr>
<tr>
<td>Strain</td>
<td>0.100</td>
<td>0.167</td>
<td>0.167</td>
<td>0.167</td>
<td>0.167</td>
<td>0.167</td>
<td>0.167</td>
<td>0.167</td>
<td>0.167</td>
</tr>
<tr>
<td>Stress MPa</td>
<td>642.5</td>
<td>642.5</td>
<td>642.5</td>
<td>642.5</td>
<td>642.5</td>
<td>642.5</td>
<td>642.5</td>
<td>642.5</td>
<td>642.5</td>
</tr>
<tr>
<td>Stress MPa at F</td>
<td>642.5</td>
<td>642.5</td>
<td>642.5</td>
<td>642.5</td>
<td>642.5</td>
<td>642.5</td>
<td>642.5</td>
<td>642.5</td>
<td>642.5</td>
</tr>
</tbody>
</table>

[Ans: a 211 GPa; b 620 GPa; c 650 MPa; d %EL = 16.75%, %AR = 49.5%; e 937 MPa; f 90 J cm\(^{-3}\); g 0.9 J cm\(^{-3}\).]

5. A design in which the primary concern is permanent deformation is termed yield limited design. Consider a panel (such as an automobile panel) which has a length \(L\), width \(W\) and depth (thickness) \(D\), and is firmly supported at the edges as depicted in Figure Q.5. When a load \(F\) is applied to the panel, say by pressing against it, the panel obviously deflects (bends). If the load is sufficient, the panel deforms plastically (permanently). The load \(F_y\) required for the panel to yield depends on the yield strength \(\sigma_y\) and, from the “Mechanics of Structures” is given by

\[
F_y = \frac{cWD^2}{L} \sigma_y
\]

where \(c\) is a numerical factor (a constant) of the order of unity that depends on whether all four edges or just two edges are supported. Its value is not essential to this design problem. (If two edges are held as in the figure, than \(c = 0.67\).)
Consider four possible candidates for an automobile panel that are listed in Table Q.5.

**a** Make a table and order the choice of materials according to least weight for the car panel given fixed panel length and width and design load $F_y$ for the panel to withstand permanent deformation. Assign an arbitrary figure of merit of unity to the aluminum alloy and determine the relative figure of merit for each material. Show the relative figures of merit in your table.

**b** Make a table and order the materials according to cost given fixed panel length and width and design load $F_y$ for the panel to withstand permanent deformation. Assign an arbitrary figure of merit of unity to the aluminum alloy and determine the relative figure of merit for each material. Show the relative figures of merit in your table.

**Table Q.5:** Properties of four candidate materials for an automobile panel.

<table>
<thead>
<tr>
<th>Material</th>
<th>Density (g cm$^{-3}$)</th>
<th>Yield strength (MPa)</th>
<th>Yesterday's price per kg</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel</td>
<td>7.9</td>
<td>400</td>
<td>$15</td>
</tr>
<tr>
<td>Al-alloy</td>
<td>2.7</td>
<td>150</td>
<td>$15</td>
</tr>
<tr>
<td>GFRP (Glass fibre reinforced polymer)</td>
<td>1.7</td>
<td>100</td>
<td>$20</td>
</tr>
<tr>
<td>Titanium</td>
<td>4.5</td>
<td>250</td>
<td>$200</td>
</tr>
</tbody>
</table>

6. For a 75% cold worked aluminum alloy, the recrystallization time is about 200 hr. at 250 °C and only 1 hr. 30 min. at 300 °C. At what temperature would the recrystallization process occur in 5 minutes?

[Ans: 334 °C ]

7. A cold worked copper component takes 45 minutes to recrystallize at 120 °C but takes 6 minutes at 170 °C. How long will the recrystallization take place at 150 °C. Can you cold work or strain harden this component at 350 °C? Why can’t you cold work Pb and Sn at room temperature?

[Ans: 12.7 min ; No as the recrystallization time is 3.7 secs; 0.60 eV]
8. **a** A brass support in the shape of a rod with a diameter 2 mm and length 500 mm has a VHN (Vickers Hardness Number) of 90. It is cold worked by permanently elongating it to 600 mm. What would be the observed yield strength and tensile strength of this component if we were to carry out a tensile test sometime after it had been plastically deformed (elongated)?

**b** Consider a brass specimen of diameter 2 mm and length 100 mm which has a VHN of 90. The specimen is deformed by a tensile tensile load of 1.1 kN. The load is removed and after a while the specimen is re-tested in a tensile test. What are the observed approximate yield strength and tensile strength?

**Bibliography**
