HIGH-FREQUENCY SMALL-SIGNAL BJT MODEL

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A pnp transistor operated in the active region in the common emitter configuration. The dc voltage across the BE junction, \( V_{BE} \), controls the current \( I_E \) and hence \( I_B \) and \( I_C \). The input current is the current that flows between \( V_{BE} \) and the base, which is \( I_B \). The output current is the current flowing between \( V_{CE} \) and the collector, which is \( I_C \).

We consider the pnp bipolar transistor depicted in Figure 1. Under normal dc operating conditions, the minority carrier (hole) concentration profile and therefore the injected minority carrier (hole) charge in the base will be constant, as shown in Figure 1. The base current simply replaces those majority carriers (electrons) continuously consumed in the recombination. If \( Q_B \) is the total amount of injected minority carrier charge in the base due to the dc voltage \( V_{BE} \), which is shown in Figure 2, then the dc base current \( I_B \) is simply \( Q_B / \tau_h \), as \( 1/\tau_h \) is the mean rate of recombination,

\[
I_B = \frac{Q_B}{\tau_h}
\]

DC base current
Suppose that we now increase $V_{BE}$ by $\delta V_{BE}$, which leads to an increase in $p_n(0)$ to $p'_n(0)$, which is shown as the dashed line in Figure 2. With more minority carriers injected, there is now an additional stored charge in the base, as indicated by the gray shaded area and labeled as $\delta Q_B$ in Figure 2. Since the stored charge, $Q_B$, in the base depends on $V_{BE}$, there is a capacitive effect appearing between the $BE$ terminals. We represent the capacitive effect across the base-emitter terminals by defining a small-signal diffusion1 (or storage) capacitance, $C_{diff}$, by

$$C_{diff} = \frac{\delta Q_B}{\delta V_{BE}}$$

Base diffusion capacitance

The stored charge, neglecting recombination, as shown in Figure 2, is

$$Q_B = \frac{1}{2} eAW_bp_n(0) = \frac{1}{2} eAW_bp_{no} \exp\left(\frac{eV_{BE}}{kT}\right)$$

Differentiating this with respect to $V_{BE}$ we obtain

$$C_{diff} = \frac{eQ_B}{kT} = \frac{e\tau_b I_B}{kT} = \frac{\tau_b eI_C}{\beta kT} = \frac{\tau_b}{r_{be}}$$

where we have used

$$r_{be} = \frac{\beta kT}{eI_E} \approx \frac{\beta kT}{eI_C}$$

This is the capacitance that has to be charged and discharged as the signal $v_{be}$ modulates $V_{BE}$. Its value is generally greater than the capacitance of the $BE$ depletion region. For example, typically, for a transistor with $\beta = 100$ and a minority carrier lifetime of 1 $\mu$s, when $I_c = 1$ mA, $r_{be} = 2.5$ k$\Omega$ and

$$C_{diff} = \frac{\tau_b}{r_{be}} = \frac{10^{-6}}{2500} = 0.4 \text{ nF}$$

There is also the capacitance of the depletion region, $C_{dep}$, between the base and the emitter. Its value for an abrupt junction was derived during the treatment of the $pn$ junction and depends on the width of the $BE$ depletion region, $W_{BE}$,

$$C_{dep} = \frac{\varepsilon A}{W_{BE}}$$

where $W_{BE}$ decreases with increasing $V_{BE}$. The total small-signal capacitance across $BE$ is therefore

$$C_{be} = C_{diff} + C_{dep}$$

The small signal equivalent circuit of the BJT must therefore also include the capacitance $C_{bc}$, as shown in Figure 3 (a). As the base-collector junction is reverse biased, there is, in addition, a depletion region capacitance between the base and the collector terminals, which is shown as $C_{bc}$ in Figure 3 (a). It is apparent that $C_{bc}$ provides a feedback path for the output current into the input, which, generally, deteriorates the gain of the BJT amplifier.

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1 The stored charge in the base is due to the diffusion of injected minority carriers in this region.
Figure 3

The structure and geometry of nearly all bipolar transistors is such that recombination invariably occurs over a region rather than at one specific location. Figure 4 shows a simplified schematic sketch of a typical pnp BJT fabricated by the commonly used planar technology. It is apparent that the device is not symmetric. The surface area of the collector junction is larger than that of the emitter junction. We define the active base region as the volume of base that contains the majority of the emitter to collector hole current, which is indicated (only roughly) in the figure. The base current has to supply electrons to the closest and farthest points of recombination, which means that it flows over the whole active region of the base, though its magnitude gets smaller farther away from the base terminal, \( B \). Since the base material has a finite resistivity, there is therefore a distributed voltage drop along the active region. In other words, the voltage across the base-emitter, \( V_{BE} \), gets smaller as we move away from \( B \). Thus the minority carrier injection and hence \( I_E \) gets less farther away from \( B \). Suppose that point \( B' \) is some mean point in the active base region, as roughly shown in Figure 4, that represents an effective or a true base point such that \( V_{BE} \) is the effective base-emitter voltage that controls the emitter current. Thus, by definition,

\[
I_E = I_{EO} \exp \left( \frac{eV_{BE}}{kT} \right)
\]

Then we can calculate (or obtain by measurement) the effective resistance, say \( r_{bb'} \), of the base region, which gives rise to the voltage drop \( V_{BE} - V_{B'E} \):

\[
V_{BE} - V_{B'E} = I_B r_{bb'}
\]
where $r_{bb'}$ is called the **base spread resistance**. It takes into account not only the distributed voltage drop in the active base region due to flow of $I_B$ but also some of the base material outside the active base to the base terminal, $B$. Figure 4 shows that $r_{bb'}$ is placed between the base terminal, $B$, and the true base point, $B'$, and has the base current, $I_B$, flowing through it.

A simplified representation of the structure of a typical $pnp$ bipolar transistor (for example, as fabricated by diffusion processes). The base current flows through a finite semiconductor region to replenish the electrons lost by recombination. It has to supply electrons to the closest and also to the farthest point in the base. Point $B'$ represents a mean point in the active base region that acts as an effective base point.

**Figure 4**

As $B'$ represents the true base point, the capacitances $C_{be}$ and $C_{bc}$ involving the active base are now between $B'$ and $E$, and $B'$ and $C$ so that they become $C_{be'}$ and $C_{bc'}$. The modified small signal equivalent circuit is shown in Figure 3 (b). The model, in addition, has a resistance $r_{ce}$ placed between the $CE$ terminals due to the following effect. Suppose that $V_{BE}$ is kept constant and $V_{CE}$ is increased by an amount $\delta V_{CE}$. This increases the reverse bias $V_{CB}$ by $\delta V_{CB}$. Consequently the base width, $W_B$, gets narrower as

**Figure 5**
shown in Figure 5 (due to the Early effect), which leads to a steeper minority carrier concentration gradient and hence to a greater collector current, say by an amount $\delta I_C$. Since $\delta V_{CE}$ leads to $\delta I_C$, the two output parameters are related just as they would be in a resistance. We define a small signal collector-emitter resistance $r_{ce}$ by

$$r_{ce} = \frac{\delta V_{CE}}{\delta I_C} \approx \frac{v_{ce}}{i_c}$$

The modulation of the base width $W_B$ by $V_{CE}$ is not very strong and hence $\delta I_C$ is generally very small. Consequently $r_{ce}$ is quite large, typically greater than 50 kΩ. It is represented as a resistance across the CE terminals of the small signal equivalent circuit as shown in Figure 3 (b).

There are other effects, usually of secondary nature, in the small signal equivalent circuit which are beyond the scope of this book. Their inclusion does not significantly affect the predictions of the model. Small-signal equivalent circuits, such as that in Figure 3 (b), are most useful in obtaining the frequency response.

For example, suppose that an ac source, $v_s$, is connected across the base-emitter (in series with a dc bias, which is not shown in small-signal equivalent circuits). We adjust $v_s$ so that the input current, $i_b$, is always constant. What is the current gain, $\beta$, at different frequencies? The impedance between $B'$ and $E$, $z_{b'e}$, is given by $r_{b'e}$ and $C_{b'e}$ in parallel. At low frequencies, we can neglect the impedance of $C_{b'e}$ so that there is only $r_{b'e}$. The low-frequency gain, $\beta$, is then

$$\beta = \frac{i_c}{i_b} = \frac{g_m v_{b'e}}{v_{b'e}} = g_m r_{b'e} = \beta_o$$

where by definition $\beta_o = g_m r_{b'e}$ and represents the low-frequency current gain.

At high frequencies, $z_{b'e}$ becomes shunted by the small impedance of $C_{b'e}$ so that

$$z_{b'e} = \frac{1}{j\omega C_{b'e}}$$

The magnitude of the current amplification at high frequencies is

$$|\beta| = \frac{i_c}{|i_b|} = \frac{g_m v_{b'e}}{v_{b'e} \left| z_{b'e} \right|} = \frac{g_m}{\omega C_{b'e}}$$

The frequency dependence of the current gain $\beta$ shown on a log-log plot. The cutoff frequency is when

$$\beta = \beta_o / \sqrt{2}.$$
We see that the current gain decreases with the frequency in the high-frequency range. At high frequencies, $C_{b'e}$ shunts $B'E$ and thereby reduces $v_{b'e}$ which results in a smaller output current, $i_c = g_m v_{b'e}$. The overall current amplification is shown in Figure 6, where the current gain exhibits a cut-off frequency at $f = f_\beta$, where its magnitude has fallen by a factor of $\sqrt{2}$. This occurs when the impedance of $C_{b'e}$ is equal to $r_{b'e}$

$$f_\beta = \frac{1}{2\pi r_{b'e} C_{b'e}}$$

### NOTATION

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>$A$</td>
<td>area</td>
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<tr>
<td>$B$</td>
<td>base terminal BJT</td>
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<tr>
<td>BJT</td>
<td>bipolar junction transistor</td>
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<tr>
<td>$C$</td>
<td>collector terminal of BJT; capacitance</td>
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<tr>
<td>CE</td>
<td>common emitter configuration</td>
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<tr>
<td>$e$</td>
<td>electronic charge ($1.602 \times 10^{-19}$ C)</td>
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<tr>
<td>$E$</td>
<td>electric field</td>
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<tr>
<td>$E$</td>
<td>emitter terminal of BJT</td>
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<tr>
<td>$f$</td>
<td>frequency</td>
</tr>
<tr>
<td>$g_m$</td>
<td>mutual transconductance</td>
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<tr>
<td>$I_B, I_C, I_E$</td>
<td>base, collector and emitter currents</td>
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<tr>
<td>$k$</td>
<td>Boltzmann’s constant ($1.381 \times 10^{-23}$ J K$^{-1}$)</td>
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<tr>
<td>$p_n(x)$</td>
<td>minority carrier concentration profile</td>
</tr>
<tr>
<td>$Q_B$</td>
<td>total injected minority carrier charge in base</td>
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<td>$R, r$</td>
<td>resistance</td>
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<tr>
<td>$r_{b'e}$</td>
<td>base spread resistance</td>
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<td>voltage</td>
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<td>$V_{CC}$</td>
<td>supply voltage</td>
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<tr>
<td>$W_B$</td>
<td>base width</td>
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<tr>
<td>$\beta$</td>
<td>low-frequency gain</td>
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<tr>
<td>$\varepsilon$</td>
<td>permittivity of a medium (C V$^{-1}$ m$^{-1}$ or F m$^{-1}$)</td>
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<tr>
<td>$\pi$</td>
<td>pi, 3.14159...</td>
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<tr>
<td>$\tau_h$</td>
<td>mean rate of recombination</td>
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<tr>
<td>$\omega$</td>
<td>angular frequency</td>
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### USEFUL DEFINITIONS

**Early effect** (Base width modulation) is the modulation of the base width by the voltage appearing across the base-collector junction. An increase in the base to collector voltage increases the collector junction depletion layer width, which results in the narrowing of the base width, hence increasing the collector current.

**Base spread resistance** ($r_{b'e}$) is an effective resistance that represents the voltage drop from the external base terminal ($B$) to the actual base point ($B'$). It is the voltage $V_{b'e}$ that controls the collector current.

**Bipolar junction transistor (BJT)** is a transistor whose normal operation is based on the injection of minority carriers from the emitter into the base region and their diffusion to the collector, where they give rise to a collector current. The voltage between the base and the emitter controls the collector current; this is the transistor action.

**Majority carriers** are electrons in an $n$-type and holes in a $p$-type semiconductor.

**Minority carriers** are electrons in a $p$-type and holes in an $n$-type semiconductor.

**pn junction** is a contact between a $p$-type and an $n$-type semiconductor. It has rectifying properties.

**Recombination** of an electron hole pair involves an electron in the conduction band (CB) falling in energy down into an empty state (hole) in the valence band (VB) to occupy it. The result is the annihilation of the electron-hole pair. Recombination is direct when the electron falls directly down into an empty state in the VB as in GaAs. Recombination is indirect if the electron is first captured locally by a defect or an impurity, called a recombination center, and from there it falls down into an empty state (hole) in the VB as in Si and Ge.